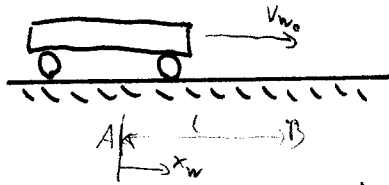


geg.: L, h, v_w ($v_w = \text{const}$)

ges.: v_{k0}



$a_w = 0$ (da v_w const)

$a_k = g = 9,81 \frac{\text{m}}{\text{s}^2}$ (us. freiem Fall)

$$v_k = \int_0^t g dt = g \cdot t + C \quad \Big|_0^t \quad C = v_{k0}$$

$$v_k = g \cdot t + v_{k0}$$

$$x_w = v_{w0} \cdot t + x_{w0} \quad (x_w(t=0) = 0 \rightarrow x_{w0} = 0)$$

$$x_w = v_{w0} \cdot t$$

$$x_k = \int_0^t v_k dt = \int_0^t (g \cdot t + v_{k0}) dt = \frac{g \cdot t^2}{2} + v_{k0} \cdot t + x_{k0} \quad | \quad x_k(t=0) = 0 \rightarrow x_{k0} = 0$$

$$x_k = \frac{g \cdot t^2}{2} + v_{k0} \cdot t$$

Bedingung:

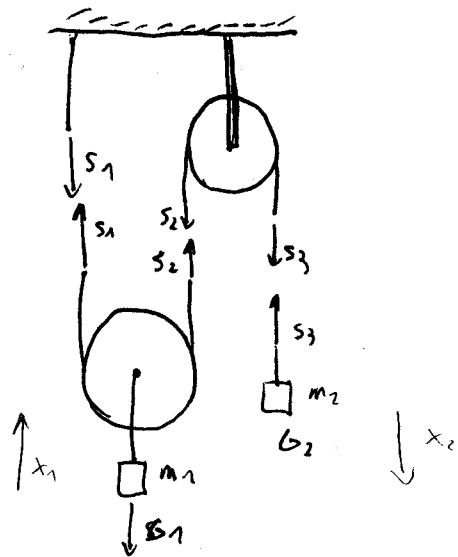
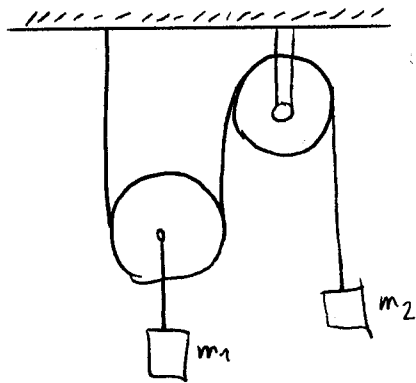
$$L = x_w(T) \quad x_k(T) = h$$

$$L = v_{w0} \cdot T \quad h = \frac{g \cdot T^2}{2} + v_{k0} \cdot T$$

$$\leadsto T = \frac{L}{v_{w0}}$$

$$\Rightarrow h = \frac{g \cdot \left(\frac{L}{v_{w0}}\right)^2}{2} + v_{k0} \cdot \frac{L}{v_{w0}} = \frac{g \cdot L^2}{2 v_{w0}^2} + \frac{v_{k0} \cdot L}{v_{w0}}$$

$$v_{k0} = \left(h - \frac{g \cdot L^2}{2 v_{w0}^2}\right) \cdot \frac{v_{w0}}{L} = \frac{h v_w}{L} - \frac{g \cdot L}{2 v_w} = \frac{v_w \cdot h}{L} \left(1 - \frac{g \cdot L^2}{2 v_w^2 h}\right)$$



Rolle 1: $m_1 \cdot \ddot{x}_1 = S_1 + S_2 - m_1 \cdot g$

Masse 2: $m_2 \cdot \ddot{x}_2 = m_2 \cdot g - S_3$

$S_1 = S_2 = S_3 = S$

Flaschenzug: $x_1 = \frac{1}{2} x_2 \quad \uparrow \quad x_2 = 2x_1$

I $m_1 \cdot \ddot{x}_1 = 2S - m_1 \cdot g$

2 · II $2m_2 \cdot \ddot{x}_2 = 2m_2 \cdot g - 2S$

$m_1 \cdot \ddot{x}_1 + 2m_2 \cdot \ddot{x}_2 = -m_1 g + 2m_2 \cdot g$

$x_2 = 2x_1$

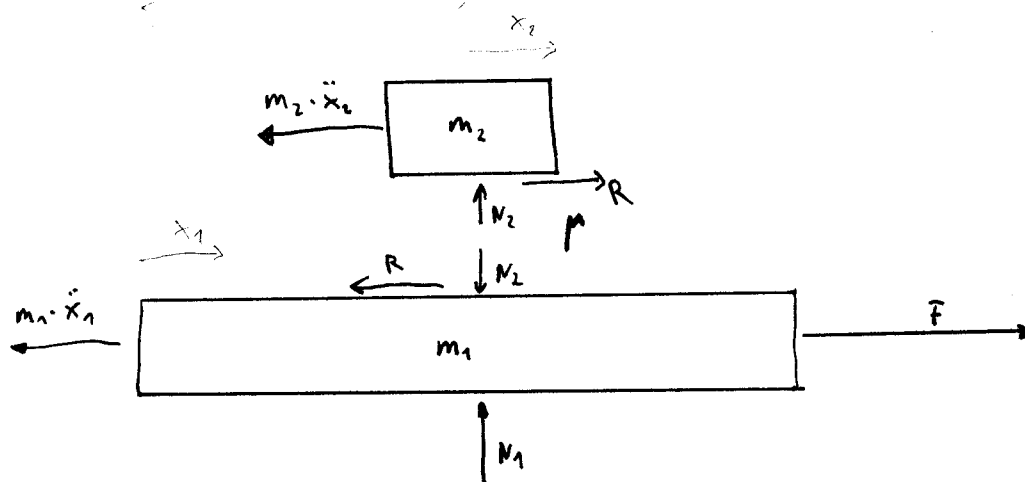
$m_1 \cdot \ddot{x}_1 + 2m_2 \cdot 2\ddot{x}_1 = -m_1 g + 2m_2 g$

$(m_1 + 4m_2) \ddot{x}_1 = (2m_2 - m_1) g$

$x_1 = \frac{2m_2 - m_1}{m_1 + 4m_2} g$

II $\rightarrow S = -m_2 \cdot \ddot{x}_2 + m_2 \cdot g$

$S = -m_2 \cdot 2 \frac{m_2 - m_1}{m_1 + 4m_2} g + m_2 g = m_2 g \left(-2 \frac{m_2 - m_1}{m_1 + 4m_2} + 1 \right)$



$$\begin{aligned} \textcircled{1} \quad & F - R - m_1 \cdot \ddot{x}_1 = 0 \\ \textcircled{2} \quad & R - m_2 \cdot \ddot{x}_2 = 0 \\ \textcircled{3} \quad & R = N_2 \cdot \mu = m_2 \cdot g \cdot \mu \end{aligned}$$

$$\textcircled{3} \text{ in } \textcircled{1} \wedge \textcircled{2}$$

$$\textcircled{1a} \quad F - m_2 \cdot g \cdot \mu - m_1 \cdot \ddot{x}_1 = 0 \quad \leadsto \quad m_1 \cdot \ddot{x}_1 = F - m_2 \cdot g \cdot \mu \quad \leadsto \quad \ddot{x}_1 = \frac{F - m_2 \cdot g \cdot \mu}{m_1}$$

$$\textcircled{2a} \quad m_2 \cdot g \cdot \mu - m_2 \cdot \ddot{x}_2 = 0 \quad \leadsto \quad m_2 \cdot \ddot{x}_2 = m_2 \cdot g \cdot \mu \quad \leadsto \quad \ddot{x}_2 = g \cdot \mu$$

$$\dot{x}_1 = \int \ddot{x}_1 dt = \frac{F - m_2 \cdot g \cdot \mu}{m_1} t + \dot{x}_{10}$$

$$\dot{x}_2 = \int \ddot{x}_2 dt = g \cdot \mu \cdot t + \dot{x}_{20}$$

$$x_1 = \int \dot{x}_1 dt = \frac{F - m_2 \cdot g \cdot \mu}{2 m_1} t^2 + \dot{x}_{10} t + x_{10}$$

$$x_2 = \int \dot{x}_2 dt = \frac{g \cdot \mu t^2}{2} + \dot{x}_{20} t + x_{20}$$

Fällt runter wenn $x_1 = L + x_2$

$$\Rightarrow \frac{F - m_2 \cdot g \cdot \mu}{2 m_1} t^2 = L + \frac{g \cdot \mu t^2}{2}$$

$$\frac{F - m_2 \cdot g \cdot \mu}{2 m_1} t^2 - \frac{g \cdot \mu t^2}{2} = L$$

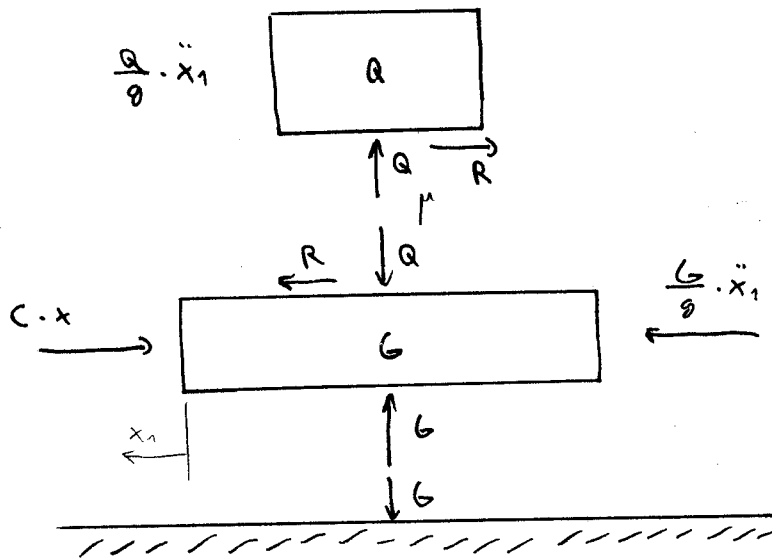
$$\left(\frac{F_1 - m_2 \cdot g \cdot \mu}{2 m_1} - \frac{g \cdot \mu}{2} \right) t^2 = L$$

$$t = \sqrt{\frac{L}{\frac{F_1}{2 m_1} - \frac{m_2 \cdot g \cdot \mu}{2 m_1} - \frac{g \cdot \mu \cdot m_1}{2 m_1}}}$$

$$t = \sqrt{\frac{2 L m_1}{F_1 - (m_2 g \mu + g \mu m_1)}}$$

$$t = \sqrt{\frac{2 L m_1}{F - \mu \cdot g (m_2 + m_1)}}$$

4.2



Q und G hier als Gewichtskräfte

Körper G: ① $-c x + R + \frac{G}{g} \cdot \ddot{x} = 0$

Körper Q: ② $\frac{Q}{g} \cdot \ddot{x} - R = 0$

Reibung: ③ $R = Q \cdot \mu$

③ in ① und ②

①a $-c x + Q \cdot \mu + \frac{G}{g} \ddot{x} = 0$

②a $\frac{Q}{g} \cdot \ddot{x} - Q \cdot \mu = 0$

$$\ddot{x} = \frac{Q \cdot \mu \cdot g}{Q}$$

2a) in 1a)

$$-c \cdot x + Q \cdot \mu + \frac{G}{g} \cdot \mu \cdot g = 0$$

$$Q \cdot \mu + G \cdot \mu = c \cdot x$$

$$M \cdot (G + Q) = c \cdot x$$

$$x = \frac{M \cdot (G + Q)}{c}$$

Energiesatz: $E_{kin} = E_{pot}$

$$\frac{1}{2} m v^2 = \frac{1}{2} c x^2$$

$$m v^2 = c x^2$$

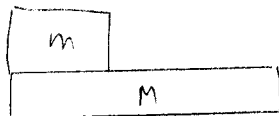
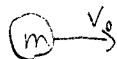
$$| m = \frac{G + Q}{g}$$

$$v^2 = \frac{c \cdot g}{G + Q} \cdot \left[\frac{M \cdot (G + Q)}{c} \right]^2$$

$$v^2 = g \cdot \frac{G + Q}{c} \cdot M^2$$

$$v = M \cdot \sqrt{\frac{g \cdot (G + Q)}{c}}$$

4.3



geg.: $v_1 = v_0$ ($e = 1$)

$$v_2 = 0$$

ges.: $w_1 = ?$
 $w_2 = ?$

Impulssatz:

$$m_1 \cdot v_0 + m_2 \cdot v_2 = w_1 \cdot m_1 + w_2 \cdot m_2$$

$$\left| \begin{array}{l} m_1 = m_2 = m \\ v_2 = 0 \end{array} \right.$$

$$m \cdot v_0 = m(w_1 + w_2)$$

$$v_0 = w_1 + w_2$$

$$w_1 = v_0 - w_2$$

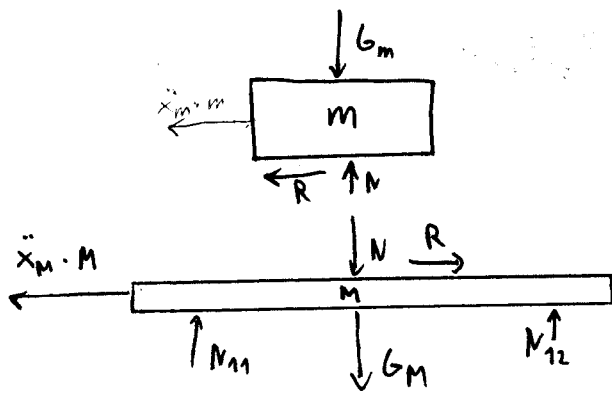
Stoßzahl:

$$e = 1 = \frac{w_2 - \overbrace{(v_0 - w_2)}^{w_1}}{v_0}$$

$$1 = \frac{w_2 - v_0 + w_2}{v_0} = \frac{2w_2 - v_0}{v_0} = \frac{v_0}{v_0} \left(\frac{2w_2}{v_0} - 1 \right)$$

$$2 = \frac{2w_2}{v_0}$$

$$v_0 = w_2 \Rightarrow v_0 = w_1 + v_0 \Rightarrow w_1 = 0$$



Klotz: ① $-\ddot{x}_m \cdot m - R = 0 \quad \Leftrightarrow \quad \ddot{x}_m = -\frac{R}{m} = -\frac{\mu \cdot g \cdot m}{m} = -\mu \cdot g$

Brett: ② $-\ddot{x}_m \cdot M + R = 0 \quad \Leftrightarrow \quad \ddot{x}_M = \frac{R}{M} = \frac{m \cdot g \cdot \mu}{M}$

③ $R = m \cdot g \cdot \mu$

$$\dot{x}_m = \int \ddot{x}_m dt = -\mu \cdot g \cdot t + \dot{x}_{m0} = -\mu \cdot g \cdot t + v_0 \quad | \dot{x}_{m0} = v_0$$

$$\dot{x}_M = \int \ddot{x}_M dt = \mu \cdot g \cdot \frac{m}{M} \cdot t + \dot{x}_{M0} \quad | \dot{x}_{M0} = 0$$

Ende des Rutschens bei $\dot{x}_m(t_x) = \dot{x}_M(t_x)$

$$-\mu \cdot g \cdot t_x + v_0 = \mu \cdot g \cdot \frac{m}{M} \cdot t_x$$

$$v_0 = \mu \cdot g \cdot \frac{m}{M} \cdot t_x + \mu \cdot g \cdot t_x$$

$$v_0 = \mu \cdot g \cdot t_x \left(\frac{m}{M} + 1 \right)$$

$$t_x = \frac{v_0}{\mu \cdot g \left(\frac{m}{M} + 1 \right)} = \frac{v_0 \cdot M}{\mu \cdot g \cdot (m + M)}$$

$$\dot{x}_M(t_x) = \mu \cdot g \cdot \frac{m}{M} \cdot t_x = \mu \cdot g \cdot \frac{m}{M} \cdot \frac{v_0 \cdot M}{\mu \cdot g \cdot (m + M)} = \frac{m \cdot v_0}{m + M}$$



$$\textcircled{1} \quad m \cdot v_1 + 5m \cdot v_2 = m \cdot w_1 + 5m \cdot w_2$$

$$m \cdot v_1 = m \cdot w_1 + 5m \cdot w_2$$

$$\textcircled{2} \quad e = \frac{w_2 - w_1}{v_1 - v_2} \quad \leadsto \quad w_2 = e \cdot v_1 + w_1$$

$$\textcircled{2} \text{ in } \textcircled{1} \quad m \cdot v_1 = m \cdot w_1 + 5m(e \cdot v_1 + w_1)$$

$$v_1 = w_1 + 5(e \cdot v_1 + w_1)$$

$$v_1 = 5e \cdot v_1 + 6w_1$$

$$w_1 = \frac{v_1 - 5e \cdot v_1}{6} = \underline{\underline{\frac{(1-5e)}{6} v_1}}$$

$$w_2 = e \cdot v_1 + \frac{v_1 - 5e \cdot v_1}{6} = v_1 \left(e + \frac{1}{6} - \frac{5}{6} e \right)$$

$$= v_1 \left(\frac{1}{6} + \frac{1}{6} e \right)$$

$$w_2 = v_1 \cdot \frac{1+e}{6}$$

Energiesatz

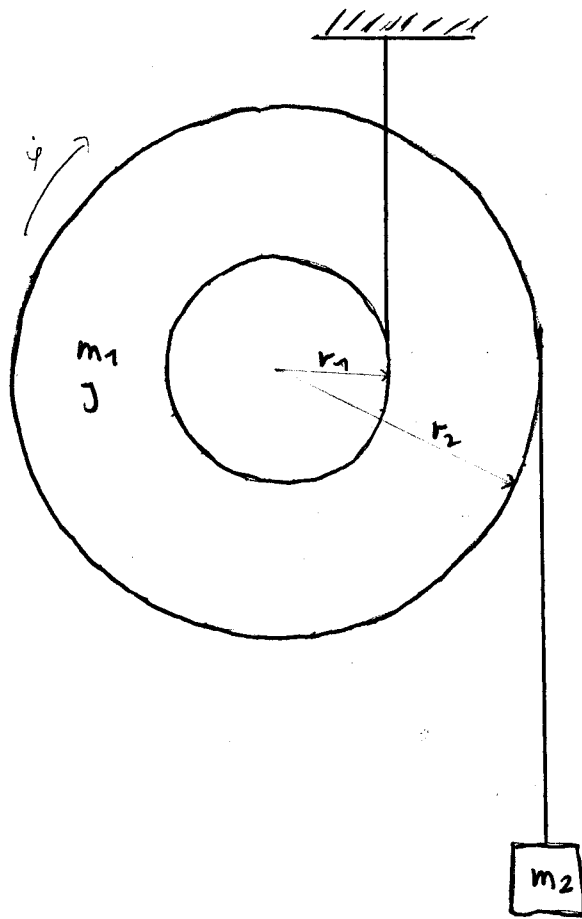
$$\frac{1}{2} 5m \cdot (w_2^2 - v_2^2) = R \cdot s$$

$$\frac{5}{2} w_2^2 m = 5m \cdot g \cdot \mu \cdot s$$

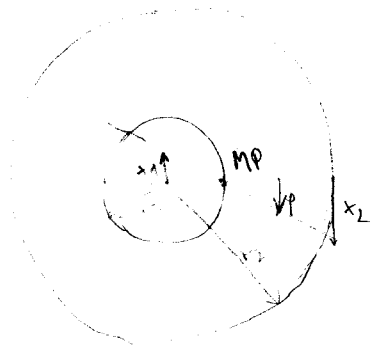
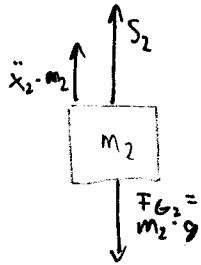
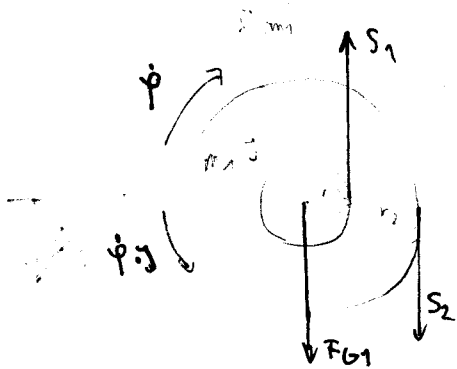
$$s = \frac{5 w_2^2}{2 \cdot 5 \cdot g \mu} = \frac{w_2^2}{2 \cdot g \mu}$$

$$s = \frac{\left(v_1 \cdot \frac{1+e}{6} \right)^2}{2 \cdot g \mu} = \frac{v_1^2 \cdot 1+e^2}{2 \cdot g \cdot \mu \cdot 36} = \underline{\underline{\frac{1+e^2}{72 \mu g}}}$$

Beispiel aus Kap. 5



Schneiden



$$\textcircled{1} m_1 \cdot \ddot{x}_1 + S_1 - S_2 - m_1 \cdot g = 0$$

$$\textcircled{2} \ddot{\varphi} \cdot J + S_1 \cdot r_1 - S_2 \cdot r_2 = 0$$

$$\textcircled{3} m_2 \cdot \ddot{x}_2 + S_2 - m_2 \cdot g = 0$$

$$\textcircled{4} \dot{x}_1 = -r_1 \cdot \dot{\varphi} \quad \Rightarrow \quad \ddot{x}_1 = -\ddot{\varphi} \cdot r_1$$

$$\textcircled{5} \dot{x}_2 = \dot{\varphi} \cdot (r_2 - r_1) \quad \Rightarrow \quad \ddot{x}_2 = \ddot{\varphi} \cdot (r_2 - r_1)$$

4 in 1 \wedge 5 in 3

$$\textcircled{1a} -m_1 \cdot \ddot{\varphi} \cdot r_1 + S_1 - S_2 - m_1 \cdot g = 0$$

$$\textcircled{2a} \ddot{\varphi} \cdot J + S_1 \cdot r_1 - S_2 \cdot r_2 = 0$$

$$\textcircled{3a} m_2 \cdot \ddot{\varphi} \cdot (r_2 - r_1) + S_2 - m_2 \cdot g = 0 \quad \leadsto \quad S_2 = m_2 \cdot g - m_2 \cdot \ddot{\varphi} \cdot (r_2 - r_1)$$

3a in 1a \wedge 3a in 2a

$$\textcircled{1b} -\ddot{\varphi} \cdot m_1 \cdot r_1 + S_1 - (m_2 \cdot g - m_2 \cdot \ddot{\varphi} \cdot (r_2 - r_1)) - m_1 \cdot g = 0$$

$$\textcircled{2b} \ddot{\varphi} \cdot J + S_1 \cdot r_1 - (m_2 \cdot g - m_2 \cdot \ddot{\varphi} \cdot (r_2 - r_1)) \cdot r_2 = 0$$

$$S_1 = \frac{[m_2 \cdot g - m_2 \cdot \ddot{\varphi} \cdot (r_2 - r_1)] \cdot r_2 - \ddot{\varphi} \cdot J}{r_1}$$

$$S_1 = \frac{m_2 \cdot g \cdot r_2}{r_1} - \frac{m_2 \cdot \ddot{\varphi} \cdot (r_2 - r_1) \cdot r_2}{r_1} - \frac{\ddot{\varphi} \cdot J}{r_1}$$

2b in 1b

$$-\ddot{\varphi} \cdot m_1 \cdot r_1 + \frac{m_2 \cdot g \cdot r_2}{r_1} - \frac{m_2 \cdot \ddot{\varphi} \cdot (r_2 - r_1) \cdot r_2}{r_1} - \frac{\ddot{\varphi} \cdot J}{r_1} - m_2 \cdot g + \ddot{\varphi} \cdot (r_2 - r_1) - m_1 \cdot g = 0$$

$$\ddot{\varphi} \left[-m_1 \cdot r_1 - \frac{m_2 \cdot (r_2 - r_1) \cdot r_2}{r_1} - \frac{J}{r_1} + m_2 \cdot (r_2 - r_1) \right] = m_1 \cdot g - \frac{m_2 \cdot g \cdot r_2}{r_1} + m_2 \cdot g$$

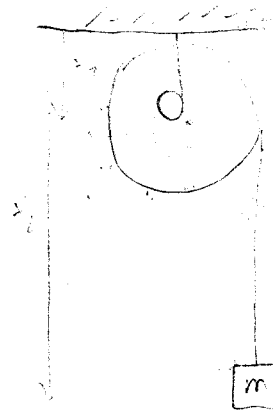
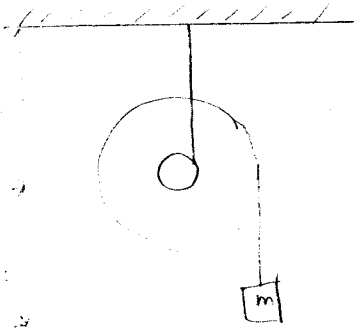
$$\ddot{\varphi} = g \cdot \frac{m_1 - m_2 \cdot \frac{r_2}{r_1} + m_2}{-m_1 \cdot r_1 - \frac{m_2 \cdot (r_2 - r_1) \cdot r_2}{r_1} - \frac{J}{r_1} + m_2 \cdot (r_2 - r_1)} \cdot \left(\frac{r_1}{r_1} \right)$$

$$\ddot{\varphi} = g \cdot \frac{m_1 \cdot r_1 - m_2 \cdot r_2 + m_2 \cdot r_1}{-m_1 \cdot r_1^2 - m_2 \cdot (r_2 - r_1) \cdot r_2 - J + m_2 \cdot (r_2 - r_1) \cdot r_1} = g \cdot \frac{m_1 \cdot r_1 - m_2 \cdot (r_2 + r_1)}{-m_1 \cdot r_1^2 - m_2 \cdot r_2^2 + m_2 \cdot r_1 \cdot r_2 - J + m_2 \cdot r_2 \cdot r_1 - m_2 \cdot r_1^2}$$

$$\ddot{\varphi} = g \cdot \frac{m_2 \cdot (r_2 + r_1) - m_1 \cdot r_1}{m_1 \cdot r_1^2 + m_2 \cdot r_2^2 - m_2 \cdot r_1 \cdot r_2 + J - m_2 \cdot r_2 \cdot r_1 + m_2 \cdot r_1^2} \quad (\text{erweitert mit } -1)$$
$$= m_2 \cdot (r_2 - r_1)^2$$

$$\ddot{\varphi} = g \cdot \frac{m_2 \cdot (r_2 + r_1) - m_1 \cdot r_1}{m_1 \cdot r_1^2 + m_2 \cdot (r_2 - r_1)^2 + J}$$

Aufgabe von vorheriger Seite mit Energieerhaltungssatz



$$E_{pot1} = -m_1 g \cdot L_1 - m_2 g \cdot (L_1 + L_2)$$

$$E_{pot2} = -m_1 g \cdot x_1 - m_2 g \cdot x_2$$

$$E_{kin1} = 0$$

$$E_{kin2} = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} J \dot{\varphi}^2$$

$$\left. \begin{aligned} x_1 &= -r_1 \cdot \varphi \\ x_2 &= (r_2 - r_1) \varphi \end{aligned} \right\}$$

$$E_{pot1} + E_{kin1} = E_{pot2} + E_{kin2}$$

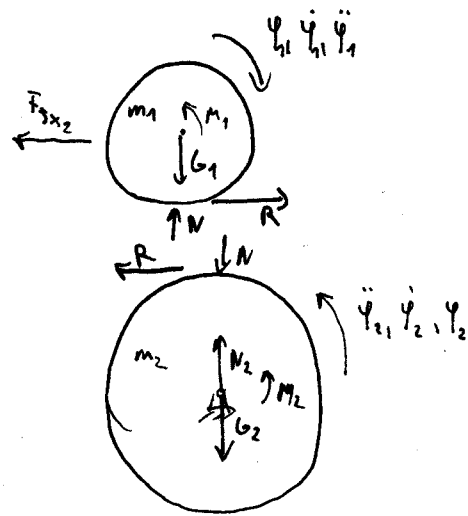
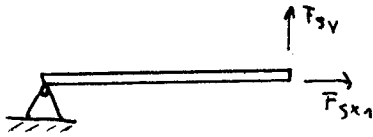
$$-m_1 g \cdot L_1 - m_2 g \cdot (L_1 + L_2) = -m_1 g \cdot x_1 - m_2 g \cdot x_2 + \frac{1}{2} (m_1 \dot{x}_1^2 + m_2 \dot{x}_2^2 + J \dot{\varphi}^2)$$

$$-m_1 g \cdot L_1 - m_2 g \cdot (L_1 + L_2) = -m_1 g \cdot r_1 \cdot \varphi - m_2 g \cdot (r_2 - r_1) \varphi + \frac{\dot{\varphi}^2}{2} \left[\frac{m_1 \cdot r_1^2 + m_2 \cdot (r_2 - r_1)^2 + J}{1} \right]$$

$$\frac{\dot{\varphi}^2}{2} = \frac{g \cdot [L_1 + m_2(L_1 + L_2)] - \varphi g (m_1 \cdot r_1 - m_2(r_2 - r_1))}{m_1 \cdot r_1^2 + m_2 (r_2 - r_1)^2 + J}$$

mit $L_1 = 0$ und $L_2 = 0$:

$$\frac{\dot{\varphi}^2}{2} = \varphi \cdot \frac{g \cdot m_2 (r_2 - r_1) - g \cdot m_1 \cdot r_1}{m_1 \cdot r_1^2 + m_2 (r_2 - r_1)^2 + J}$$



$$F_{3y} = 0$$

$$F_{5x1} - F_{5x2} = 0$$

$$1) \quad N_0 = m_1 \cdot g$$

$$2) \quad R = \mu \cdot N_1$$

$$3) \quad M_1 = R \cdot r_1$$

$$4) \quad M_2 = R \cdot r_2$$

$$5) \quad M_1 = -J_1 \cdot \ddot{\psi}_1 \quad \leadsto \quad \ddot{\psi}_1 = -\frac{M_1}{J_1} \quad \dot{\psi}_1 = -\frac{M_1}{J_1} t + \dot{\psi}_{10}$$

$$6) \quad M_2 = J_2 \cdot \ddot{\psi}_2 \quad \leadsto \quad \ddot{\psi}_2 = \frac{M_2}{J_2} \quad \dot{\psi}_2 = \frac{M_2}{J_2} \cdot t + \dot{\psi}_{20}$$

1 in 2, 3 in 5, 4 in 6

$$2a) \quad R = m_1 \cdot g \cdot \mu$$

$$5a) \quad \dot{\psi}_1 = -\frac{R \cdot r_1}{J_1} t + \dot{\psi}_{10}$$

$$6a) \quad \dot{\psi}_2 = \frac{R \cdot r_2}{J_2} \cdot t$$

2a in 5a und 6a

$$5b) \quad \dot{\psi}_1 = -\frac{m_1 \cdot g \cdot \mu \cdot r_1 \cdot t}{J_1} + \dot{\psi}_{10} = -\frac{2 m_1 \cdot g \cdot \mu \cdot r_1 \cdot t}{m_1 \cdot r_1^2} + \dot{\psi}_{10}$$

$$6b) \quad \dot{\psi}_2 = \frac{m_1 \cdot g \cdot \mu \cdot r_2 \cdot t}{J_2} = \frac{2 m_1 \cdot g \cdot \mu \cdot r_2 \cdot t}{m_2 \cdot r_2^2}$$

Bedingung: $\dot{\psi}_1 \cdot r_1 = \dot{\psi}_2 \cdot r_2$ bei $t = t_x$

$$\dot{\psi}_{10} \cdot r_1 - 2 g \mu t_x = \frac{2 m_1 g \mu t_x}{m_2}$$

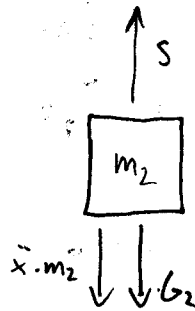
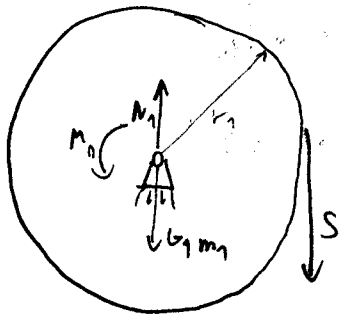
$$\frac{r \cdot \dot{\varphi}_0}{t_x} = \frac{2\mu g m_1}{m_2} + 2\mu g = 2\mu g \left(\frac{m_1}{m_2} + 1 \right)$$

$$\underline{\underline{t_x = \frac{r_1 \cdot \dot{\varphi}_0}{2\mu g \left(\frac{m_1}{m_2} + 1 \right)}}}$$

t_x einsetzen in $\dot{\varphi}_1$ und $\dot{\varphi}_2$

$$\dot{\varphi}_1 = - \frac{2\mu g}{r_1} \cdot \frac{r_1 \dot{\varphi}_0}{2\mu g \left(\frac{m_1}{m_2} + 1 \right)} + \dot{\varphi}_0 = \dot{\varphi}_0 - \frac{\dot{\varphi}_0}{\frac{m_1}{m_2} + 1}$$

$$\dot{\varphi}_2 = \frac{2\mu g m_1}{r_2 m_2} \cdot \frac{r_1 \dot{\varphi}_0}{2\mu g \left(\frac{m_1}{m_2} + 1 \right)} = \frac{m_1 \cdot r_1 \cdot \dot{\varphi}_0}{r_2 m_2 \left(\frac{m_1}{m_2} + 1 \right)}$$



1) $G_2 = m_2 \cdot g$

2) $S - G_2 - \ddot{x} \cdot m_2 = 0$

3) $\ddot{\varphi} \cdot r = \ddot{x} \quad \Rightarrow \quad \ddot{\varphi} = \frac{\ddot{x}}{r}$

4) $M_0 - S \cdot r_1 = J \cdot \ddot{\varphi}$

5) $J = \frac{1}{2} m_1 r^2$

1 in 2 3 in 4 5 in 4

2a) $S - m_2 g - \ddot{x} \cdot m_2 = 0$

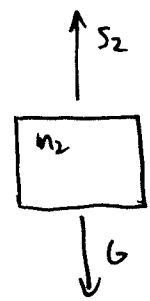
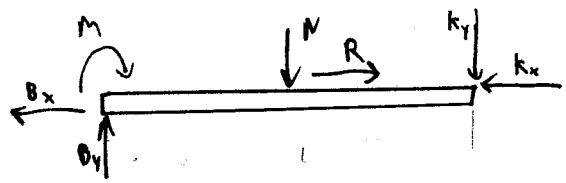
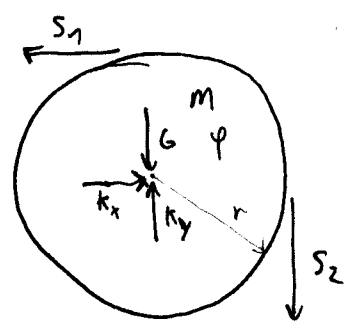
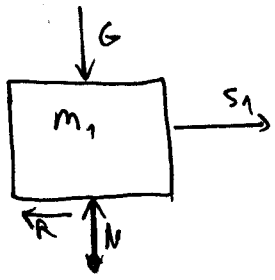
$\Rightarrow S = m_2 g + \ddot{x} \cdot m_2 \Rightarrow S = m_2 (g + a) \quad |\ddot{x} = a$

4a) $M_0 - S \cdot r_1 = \frac{1}{2} m_1 r^2 \cdot \frac{\ddot{x}}{r}$

2a in 4a

4b) $M_0 - m_2 (g + a) \cdot r = \frac{1}{2} m_1 r^2 \cdot \frac{a}{r}$

$M_0 = \frac{m_1}{2} \cdot a + m_2 \cdot r \cdot (g + a)$



- 1) $\ddot{x}_1 \cdot m = S_1 - R$
- 2) $0 = \cancel{G} - N \leadsto N = G \Rightarrow N = m \cdot g$
- 3) $\ddot{x}_2 \cdot m = G - S_2$
- 4) $\ddot{\varphi} \cdot J = r(S_2 - S_1)$
- 5) $R = \mu \cdot N$
- 6) $0 = k_y - \cancel{G} - S_2 \leadsto k_y = G + S_2$
- 7) $0 = k_x - S_1 \leadsto k_x = S_1$
- 8) $\dot{x}_2 = \dot{\varphi} \cdot r = \dot{x}_1 = \dot{x} \Rightarrow \ddot{\varphi} \cdot r = \ddot{x} \leadsto \ddot{\varphi} = \frac{\ddot{x}}{r}$

2 in 3, 5, 6 1, 8 in 1, 3, 4

- 1a) $\ddot{x} \cdot m = S_1 - R \leadsto S_1 = \ddot{x} m + R$
- 3a) $\ddot{x} \cdot m = m \cdot g - S_2 \leadsto S_2 = m \cdot g - \ddot{x} m$
- 4a) $\frac{\ddot{x}}{r} \cdot J = r(S_2 - S_1)$
- 5a) $R = \mu \cdot m \cdot g$
- 6a) $0 = k_y - m \cdot g - S_2$
- 7a) $S_1 = k_x$

5 in 1 dann 1 in 4, 3 $J = \frac{1}{2} m r^2$

$$\frac{\ddot{x}}{r} \frac{1}{2} m r^2 = r(m \cdot g - \ddot{x} m - \ddot{x} m - \mu \cdot m \cdot g)$$

$$\ddot{x} \frac{1}{2} m = m \cdot g - \ddot{x} m - \ddot{x} m - \mu \cdot m \cdot g$$

$$\frac{1}{2} \ddot{x} = g - \ddot{x} - \ddot{x} + \mu \cdot g = g(1 - \mu) - 2\ddot{x}$$

$$\ddot{x} \left(\frac{1}{2} + \frac{4}{2} \right) = g(1 - \mu) = \frac{5}{2} \ddot{x}$$

$$S_1 = \frac{2}{5}g(1-\mu) \cdot m + m \cdot g \cdot \mu$$

$$S_1 = m \cdot g \left(\frac{2}{5}(1-\mu) + \mu \right)$$

$$S_1 = m \cdot g \left(\frac{2}{5} - \frac{2}{5}\mu + \frac{5}{5}\mu \right)$$

$$S_1 = m \cdot g \frac{2+3\mu}{5}$$

$$S_2 = m \cdot g - \ddot{x} \cdot m$$

$$\ddot{x} = \frac{2}{5}g(1-\mu)$$

$$S_2 = m \cdot g - \frac{2}{5}g(1-\mu)m$$

$$S_2 = m \cdot g \left(1 - \frac{2}{5}(1-\mu) \right)$$

$$S_2 = m \cdot g \left(\frac{5}{5} - \frac{2}{5} + \frac{2}{5}\mu \right)$$

$$S_2 = m \cdot g \left(\frac{3+2\mu}{5} \right)$$

$$K_x = S_1 = m \cdot g \frac{2+3\mu}{5}$$

$$K_y = N + S_2 = m \cdot g + m \cdot g \left(\frac{3+2\mu}{5} \right)$$

$$K_y = m \cdot g \left(\frac{8}{5} + \frac{2}{5}\mu \right)$$

$$B_x + K_x - R = 0$$

$$B_x = R - K_x$$

$$B_x = m \cdot g \cdot M - m \cdot g \cdot \frac{2+3\mu}{5}$$

$$B_x = m \cdot g \left(\frac{5}{5}M - \frac{2}{5} - \frac{3}{5}\mu \right)$$

$$B_x = m \cdot g \left(\frac{2M-2}{5} \right)$$

$$B_x = \frac{2}{5}m \cdot g (\mu - 1)$$

$$B_y = N + K_y$$

$$B_y = m \cdot g + m \cdot g \left(\frac{8+2\mu}{5} \right)$$

$$B_y = m \cdot g \left(1 + \frac{8}{5} + \frac{2}{5}\mu \right)$$

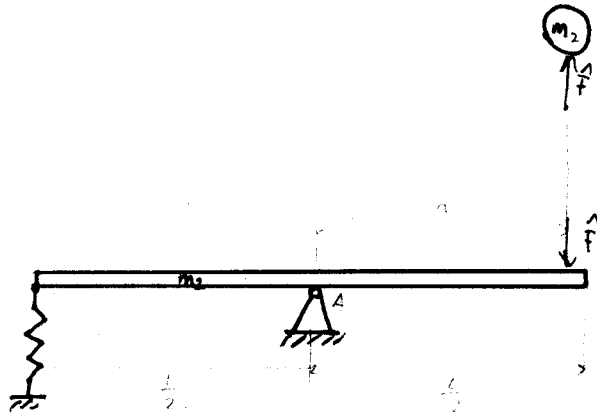
$$B_y = m \cdot g \left(\frac{13+2\mu}{5} \right)$$

$$M + N \cdot x + K_y \cdot L = 0$$

$$M = -N \cdot x - K_y \cdot L$$

$$M = -m \cdot g \cdot x - m \cdot g \left(\frac{8}{5} + \frac{2}{5}\mu \right) L$$

$$M = -m \cdot g \left(x + L \frac{8+2\mu}{5} \right)$$



- 1) $m_1 (\dot{x}_{1h} - \dot{x}_{1v}) = -\hat{F}$
- 2) $J(\dot{\varphi}_h - \dot{\varphi}_v) = a \cdot \hat{F}$
- 3) $\dot{x}_{2h} - \dot{x}_{1h} = e(\dot{x}_{1v} - \dot{x}_{2v}) = 0 \quad \Rightarrow \quad \dot{x}_2 = \dot{x}_{1h}$
- 4) $\dot{x}_{2h} = a \cdot \dot{\varphi}$

$$1a) \quad m_1 (\dot{x}_{1h} - \dot{x}_{1v}) = -\hat{F}$$

$$2a) \quad \hat{F} = \frac{J \cdot \dot{\varphi}}{a}$$

$$3a) \quad \dot{x}_{1h} = a \cdot \dot{\varphi}$$

$$m_1 (a \cdot \dot{\varphi} - \dot{x}_{1v}) = -\frac{J \dot{\varphi}}{a}$$

$$|\dot{x}_{1v} = v_0$$

$$m_1 a^2 \dot{\varphi} - a m_1 v_0 + J \cdot \dot{\varphi} = 0$$

$$\dot{\varphi} (m_1 a^2 + J) = a v_0 m_1$$

$$\dot{\varphi} = \frac{a v_0 m_1}{m_1 a^2 + J} = \frac{a v_0 m_1}{m_1 a^2 + \frac{1}{12} m_2 l^2} = \frac{a v_0}{a^2 + \frac{1}{12} \frac{m_2}{m_1} l^2}$$

Energiesatz

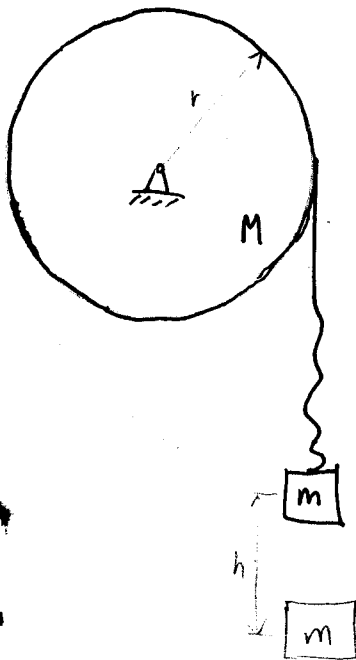
$$m \cdot g \cdot \frac{a}{L} \cdot x_{\max} + \frac{1}{2} J \dot{\varphi}^2 + \frac{1}{2} m_1 a^2 \dot{\varphi}^2 = \frac{1}{2} c x_{\max}^2$$

$$\frac{4 m g a x_{\max}}{c L} + \frac{J \dot{\varphi}^2}{c} + \frac{m_1 \cdot a^2 \dot{\varphi}^2}{c} = x_{\max}^2$$

$$0 = x_{\max}^2 - \frac{4 m g a x_{\max}}{c L} - \frac{\dot{\varphi}^2}{c} (J + m_1 a^2)$$

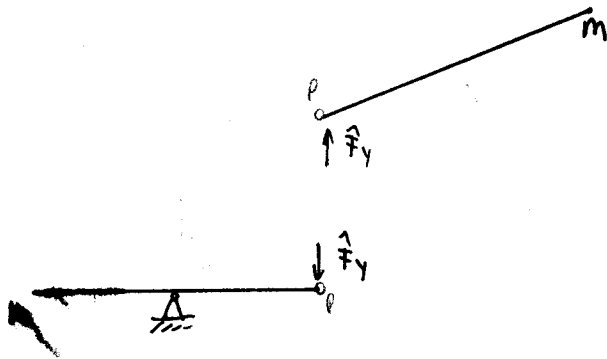
\Rightarrow p-q-Formel

5.16



- 1) $\dot{x}_{mV} = \sqrt{2gh}$
- 2) $\dot{x}_{MPV} = 0$
- 3) $\dot{\varphi}_{MN} \cdot r = \dot{x}_{MPN}$
- 4) $J(\dot{\varphi}_{MN} - \dot{\varphi}_{MV}) = r \cdot \hat{F}$
- 5) $m(\dot{x}_{mV} - \dot{x}_{mV}) = -\hat{F}$
- 6) $e = -\frac{\dot{x}_{MPN} - \dot{x}_{MPN}}{\dot{x}_{MPV} - \dot{x}_{MPV}} = 1$
- 7) $J = \frac{1}{2} M r^2$

5.17



vor Stoß

nach Stoß

$\dot{x}_{1V} = 0$

$\dot{x}_{2N} = ?$

$\dot{y}_{1V} = v_1$

$\dot{y}_{1N} = ?$

$\dot{\varphi}_{1V} = 0$

$\dot{\varphi}_{1N} = ?$

$\dot{x}_{2V} = 0$

$\dot{x}_{2N} = 0$

$\dot{y}_{2V} = 0$

$\dot{y}_{2N} = ?$

$\dot{\varphi}_{2V} = 0$

$\dot{\varphi}_{2N} = ?$

1) $m(\dot{x}_{1N} - \dot{x}_{1V}) = 0$

2) $m(\dot{y}_{1N} - \dot{y}_{1V}) = -\hat{F}_y$

3) $J(\dot{\varphi}_{1N} - \dot{\varphi}_{1V}) = \hat{F}_y \cdot L$

4) $m(\dot{x}_{2N} - \dot{x}_{2V}) = 0$

5) $m(\dot{y}_{2N} - \dot{y}_{2V}) = \hat{F}_y$

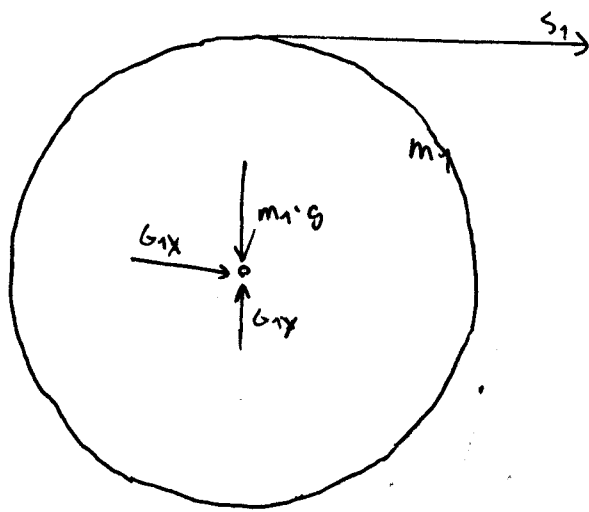
6) $J(\dot{\varphi}_{2N} - \dot{\varphi}_{2V}) = \hat{F}_y \cdot L$

7) $e = 1 = -\frac{\dot{y}_{2N} - \dot{y}_{2N}}{\dot{y}_{2V} - \dot{y}_{2V}}$

8) $\dot{\varphi}_{1N} \cdot L \cdot \cos \beta = \dot{y}_{1N}$

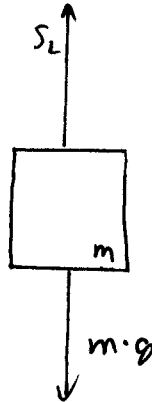
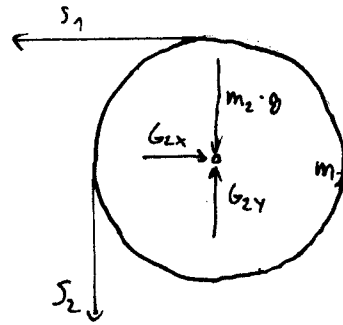
9) $J = \frac{1}{3} m L^2$

10) $\dot{y}_{2N} = \dot{\varphi}_{2N} \cdot L$



$$J_1 = \frac{1}{2} m_1 \cdot r^2$$

$$J_2 = \frac{1}{2} m_2 r^2$$



- 1) $S_1 + G_{1x} = 0$ uninteressant
- 2) $G_{1y} - m_1 \cdot g = 0$
- 3) $M_0 - S_1 \cdot r_1 = \ddot{\varphi}_1 \cdot J_1 \quad \leadsto \quad \frac{M_0 - \ddot{\varphi}_1 \cdot J_1}{r_1} = S_1$
- 4) $S_1 \cdot r_2 + S_2 \cdot r_2 = J_2 \cdot \ddot{\varphi}_2$
- 5) $G_{2x} - S_1 = 0$ uninteressant
- 6) $G_{2y} - m_2 \cdot g + S_2 = 0$
- 7) $m \cdot g - S_2 = \ddot{x} \cdot m$
- 8) $\dot{\varphi}_1 \cdot r_1 = \dot{\varphi}_2 \cdot r_2 \quad \rightarrow \quad \ddot{\varphi}_1 \cdot r_1 = \ddot{\varphi}_2 \cdot r_2$
- 9) $\dot{\varphi}_2 \cdot r_2 = \dot{x} \quad \rightarrow \quad \ddot{\varphi}_2 \cdot r_2 = \ddot{x}$

3 in 4 (\rightarrow nach S_1 umgestellt)

$$4a) \quad \frac{M_0 - \ddot{\varphi}_1 \cdot J_1}{r_1} \cdot r_2 + S_2 \cdot r_2 = J_2 \cdot \ddot{\varphi}_2$$

$$7a) \quad S_2 = m \cdot g - \ddot{x} \cdot m$$

$$8a) \quad \ddot{\varphi}_2 = \ddot{\varphi}_1 \cdot \frac{r_1}{r_2}$$

$$9a) \quad \ddot{\varphi}_2 \cdot r_2 = \ddot{x}$$

7a in 4a

8a in 9a, 4a

$$4b) \quad \frac{M_0 - \ddot{\varphi}_1 \cdot J_1}{r_1} \cdot r_2 + (m \cdot g - \ddot{x} \cdot m) r_2 = J_2 \cdot \ddot{\varphi}_1 \cdot \frac{r_1}{r_2}$$

$$9b) \quad \ddot{\varphi}_1 \cdot \frac{r_1}{r_2} \cdot r_2 = \ddot{x}$$

9b in 4b

$$4b) \frac{M_0 - \ddot{\varphi}_1 J_1}{r_1} \cdot r_2 + (m \cdot g - \bar{x} m) r_2 = J_2 \cdot \ddot{\varphi}_1 \frac{r_1}{r_2}$$

$$4c) \frac{M_0 - \ddot{\varphi}_1 J_1}{r_1} \cdot r_2 + (m \cdot g - m \ddot{\varphi}_1 \cdot r_1) r_2 = J_2 \cdot \ddot{\varphi}_1 \cdot \frac{r_1}{r_2}$$

$$-\frac{r_2 \ddot{\varphi}_1 J_1}{r_1} + \frac{M_0}{r_1} \cdot r_2 + m \cdot g \cdot r_2 - m \ddot{\varphi}_1 r_1 \cdot r_2 - J_2 \cdot \ddot{\varphi}_1 \cdot \frac{r_1}{r_2} = 0$$

$$\frac{M_0}{r_1} \cdot r_2 + m g r_2 = \ddot{\varphi}_1 m r_1 r_2 + \ddot{\varphi}_1 J_2 \cdot \frac{r_1}{r_2} + \frac{\varphi_1 J_1 r_2}{r_1}$$

$$\frac{M_0}{r_1} \cdot r_2 + m \cdot g \cdot r_2 = \ddot{\varphi}_1 \left(m \cdot r_1 \cdot r_2 + J_2 \cdot \frac{r_1}{r_2} + \frac{J_1 \cdot r_2}{r_1} \right)$$

$$\ddot{\varphi}_1 = \frac{r_2 \left(\frac{M_0}{r_1} + m \cdot g \right)}{m \cdot r_1 r_2 + J_2 \cdot \frac{r_1}{r_2} + \frac{J_1 \cdot r_2}{r_1}}$$

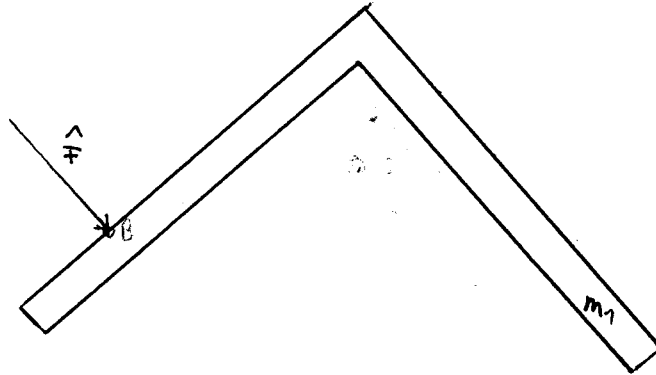
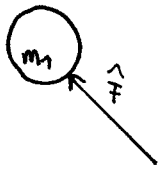
$$J_1 = \frac{1}{2} m_1 r_1^2$$

$$J_2 = \frac{1}{2} m_2 \cdot r_2^2$$

$$\ddot{\varphi}_1 = \frac{r_2 \left(\frac{M_0}{r_1} + m \cdot g \right)}{m \cdot r_1 r_2 + \frac{1}{2} m_2 r_2^2 \cdot \frac{r_1}{r_2} + \frac{1}{2} m_1 r_1^2 \frac{r_2}{r_1}}$$

$$\ddot{\varphi}_1 = \frac{r_2 \left(\frac{M_0}{r_1} + m \cdot g \right)}{r_1 r_2 \left[m + \frac{1}{2} (m_2 + m_1) \right]}$$

$$\ddot{\varphi}_1 = \frac{r_2 + m \cdot g}{r_1 \left[m + \frac{1}{2} (m_1 + m_2) \right]} = \frac{M_0 + m \cdot g \cdot r_1}{r_1^2 \left[m + \frac{1}{2} (m_1 + m_2) \right]} = \underline{\underline{\frac{2 (M_0 + m \cdot g \cdot r_1)}{r_1^2 (2m + m_1 + m_2)}}}$$

Vor Stoß

$$\dot{x}_{1V} = v_0 \cdot \cos 45^\circ$$

$$\dot{y}_{1V} = v_0 \cdot \sin 45^\circ$$

$$\dot{x}_{2V} = 0$$

$$\dot{y}_{2V} = 0$$

$$\dot{\varphi} = 0$$

nach Stoß

$$\dot{x}_{1n}, \dot{y}_{1n}, \dot{x}_{2n}, \dot{y}_n, \dot{\varphi}_n$$

$$S\left(\frac{L}{4} \mid \frac{L}{2}\right)$$

$$J = \frac{5}{24} m_2 L^2$$

$$1) m_1 (\dot{x}_{1n} - \dot{x}_{1V}) = -\hat{F}$$

$$2) m_1 (\dot{y}_{1n} - \dot{y}_{1V}) = 0 \quad \leadsto \quad \dot{y}_n = \dot{y}_{1V}$$

$$3) m_2 (\dot{x}_{2n} - 0) = \hat{F}$$

$$4) m_2 (\dot{y}_{2n} - 0) = 0 \quad \leadsto \quad \dot{y}_{2n} = 0$$

$$5) J (\dot{\varphi}_n - 0) = -\hat{F} \cdot \gamma_S$$

$$6) e = -\frac{\dot{x}_{1n} - (\dot{x}_{2n} - \frac{L}{2} \dot{\varphi}_n)}{v_0 \cdot \cos 45^\circ - 0}$$

3b in 4b

$$\left(\dot{\varphi}_n \sqrt{2} b - \frac{e v_0}{\sqrt{2}} - \frac{v_0}{\sqrt{2}} \right) m_1 = - \frac{\varphi_n m_2 a^2}{2 b \sqrt{2}}$$

$$\dot{\varphi}_n = \frac{v_0}{2b} \left(\frac{e+1}{1 + \frac{m_2 a^2}{m_1 4 b^2}} \right)$$
