(4-1)

$$p_1 = p_0 + \rho_1 \cdot g \cdot h_1 + \rho_2 \cdot g \cdot h_2 + \rho_3 \cdot g \cdot h_3$$

$$= 10^5 \frac{N}{m^2} + 790 \frac{kg}{m^3} \cdot 9{,}81 \frac{m}{s^2} \cdot 2{,}5\,m$$

$$+ 880 \frac{kg}{m^3} \cdot 9{,}81 \frac{m}{s^2} \cdot 1{,}2\,m$$

$$+ 660 \frac{kg}{m^3} \cdot 9{,}81 \frac{m}{s^2}\, 2{,}8\,m$$

$$= 14786 \frac{N}{m^2}$$

$$= 1{,}4786\ bar$$

$$p_{A,\,links} = p_{A,\,mitte} = p_{A,\,rechts}$$

$$p_0 + \rho_2 \cdot g \cdot 3L + \rho_1 \cdot g \cdot L = p_0 + \rho_1 \cdot L \cdot g = p_0 + \rho_3 \cdot g \cdot 3L$$

$$\rho_2 \cdot 3 + \rho_1 = \rho_1 = \rho_3\, 3$$

$$\frac{3}{2}\rho_2 = \rho_1 = 3 \cdot \rho_3$$

(4-3)

$$F = \rho \cdot g \cdot h_s \cdot A = 1000 \frac{kg}{m^3} \cdot 9{,}81 \frac{m}{s^2} \cdot 1{,}5\,m \cdot (1m)^2 \cdot \frac{\pi}{4}$$

$$= 11557{,}1\,N$$

$$I_s = \frac{\pi \cdot D^4}{64} \qquad z_s = \frac{h}{\sin \alpha}$$

$$e = \frac{I_s}{z_s \cdot A} = \frac{\pi \cdot D^4}{64} \cdot \frac{\sin \alpha}{h} \cdot \frac{4}{D^2 \cdot \pi} = \frac{D^2}{16} \cdot \frac{\sin \alpha}{h}$$

$$= \frac{(1m)^2}{16} \cdot \frac{\sin 60^\circ}{1{,}5\,m} = 0{,}036\,m$$

$$M = F \cdot e = 11557{,}1\,N \cdot 0{,}036\,m = 417\,Nm$$

$$F_x = \rho \cdot g \cdot h_s \cdot A = \rho \cdot g \cdot h_s \cdot \pi \cdot R^2 \qquad (a)$$

$$= 1000 \frac{kg}{m^3} \cdot 9{,}81 \frac{m}{s^2} \cdot 1{,}5\,m \cdot \pi \cdot (1\,m)^2 = 46228{,}5\,N_A$$

$$F_z = \rho \cdot g \cdot V = \rho \cdot g \cdot \frac{4}{3} \pi R^3 \cdot \frac{1}{2} = \rho \cdot g \cdot \frac{2}{3} \pi R^3 \qquad (b)$$

$$= 1000 \frac{kg}{m^3} \cdot 9{,}81 \frac{m}{s^2} \cdot \frac{2}{3} \cdot \pi \cdot (1\,m)^3 = -20546{,}0\,N_B$$

$$F = \sqrt{F_x^2 + F_z^2} = 50588{,}7\,N \qquad (c)$$

$$\alpha = \arctan\left(\frac{F_z}{F_x}\right) = \arctan\left(\frac{20546{,}0}{46228{,}5}\right) = 24^\circ \qquad (d)$$

Die Wirkungslinien aller Kräfte die an einer Kugel (e)

verlaufen durch den Mittelpunkt d. Kugel.

(4 - 5)

(a) $F_x = \rho g z_s \cdot A_x \qquad A_x = 2 \cdot R \cdot b$

$$F_{xo} = \rho g (H-R)\, 2 R b$$

$$F_{xu} = \rho g (H+R)\, 2 R b$$

$$M = F_{xu} \cdot R - F_{xo} \cdot R$$

$$= \rho g (H+R) 2 R b \cdot R - \rho g (H-R) 2 R b \cdot R$$

$$= \rho g H 2 R^2 b + \rho g 2 R^3 b - \rho g H 2 R^2 b + \rho g 2 R^3 b$$

$$= 4 \rho g b R^3$$

(b) $c_M = 4$

$$F_{Klappe} = \rho_F \cdot g \cdot h_s \cdot A = \rho_F \cdot g \cdot \left(h + \frac{a}{2}\right) a^2$$

$$= 1000 \frac{kg}{m^3} \cdot 9{,}81 \frac{m}{s^2} \cdot \left(11{,}67 \cdot 10^{-2} m + \frac{0{,}1 m}{2}\right)(0{,}1 m)^2$$

$$= 16{,}35 N$$

$$e = \frac{I_s}{z_s \cdot A} = \frac{a^4}{12 \cdot \left(h + \frac{a}{2}\right) \cdot a^2} = \frac{a^2}{12\left(\frac{14}{12}a + \frac{a}{2}\right)} = \frac{a}{20}$$

$$= \frac{0{,}1}{20} m = 5 \cdot 10^{-3} m$$

$$F_{Auftrieb} = \rho_F \cdot g \cdot V = \rho_F \cdot g \cdot \frac{4}{3} d^3 \frac{\pi}{8}$$

$$= 1000 \frac{kg}{m^3} \cdot 9{,}81 \frac{m}{s^2} \cdot \frac{4}{3} (0{,}1 m) d^3 \frac{\pi}{8}$$

$$= 5{,}14 N$$

$$F_G = \rho_K \cdot g \cdot V = 0{,}01 \cdot \rho_F \cdot g \cdot \frac{4}{3} \cdot (0{,}1 m)^3 \frac{\pi}{8}$$

$$= 0{,}01 \cdot 1000 \frac{kg}{m^3} \cdot 9{,}81 \frac{m}{s^2} \cdot (0{,}1 m)^3 \cdot \frac{\pi}{8}$$

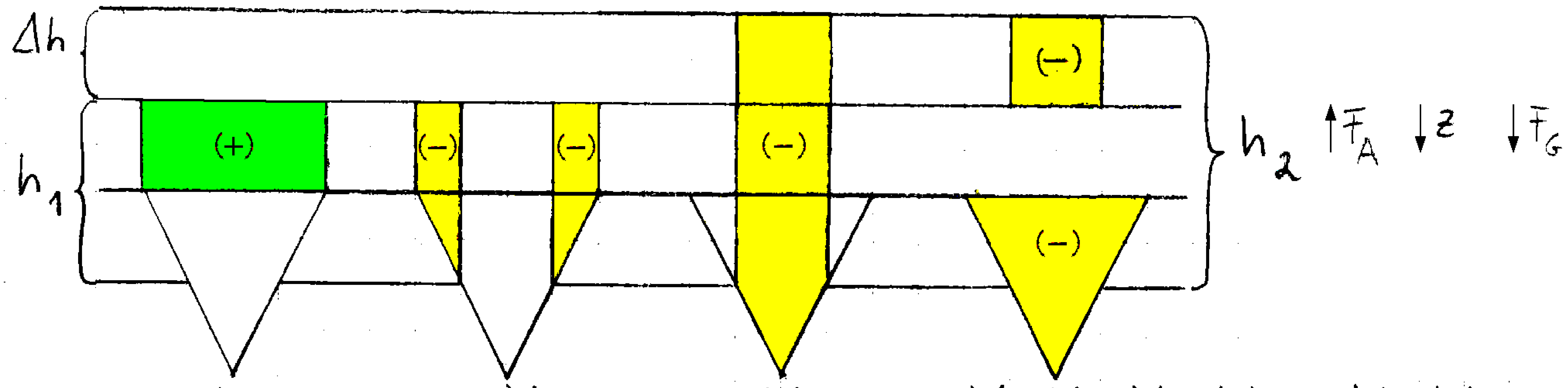
$$= 51{,}37 \cdot 10^{-3} N$$

$$\sum M_i = F_K \cdot \left(e + \frac{a}{2}\right) - F_A \cdot l + F_G \cdot l = 0$$

$$l = \frac{-F_K \cdot \left(e + \frac{a}{2}\right)}{-F_A + F_G} = \frac{-16{,}35 N \cdot \left(5 \cdot 10^{-3} m + \frac{0{,}1 m}{2}\right)}{-5{,}14 N + 51{,}37 \cdot 10^{-3} N}$$

$$= 17{,}68 cm$$

(4-7)



$$V = V_1 - V_2 - V_3 = -V_z - V_K$$

$$F_A = \rho_W g(-V_z - V_K) \qquad F_G = \rho_K g V_K \qquad \Sigma F = 0$$

$$\rho_W g(-V_z - V_K) = -\rho_K g V_K \qquad \Sigma F = F_A + F_G = 0$$

$$-V_z - V_K + \frac{\rho_K}{\rho_W} V_K = 0 \qquad \Rightarrow \quad F_A = -F_G$$

$$V_z + \left(1 - \frac{\rho_K}{\rho_W}\right) V_K = 0$$

$$\frac{V_z}{V_K} = \frac{\rho_K}{\rho_W} - 1$$

$$\frac{\frac{\pi}{4} d^2 \cdot \Delta h}{\frac{\pi}{12} D^2 \cdot h_K} = \frac{11 \rho_W}{\rho_W} - 1$$

$$3\left(\frac{d}{D}\right)^2 \frac{\Delta h}{h_K} = 11 - 1$$

$$3\left(\frac{1}{2}\frac{D}{D}\right)^2 \frac{\Delta h}{D} = 10$$

$$D = \frac{3}{40} \cdot 1\,m = 0{,}075\,m$$

(b)

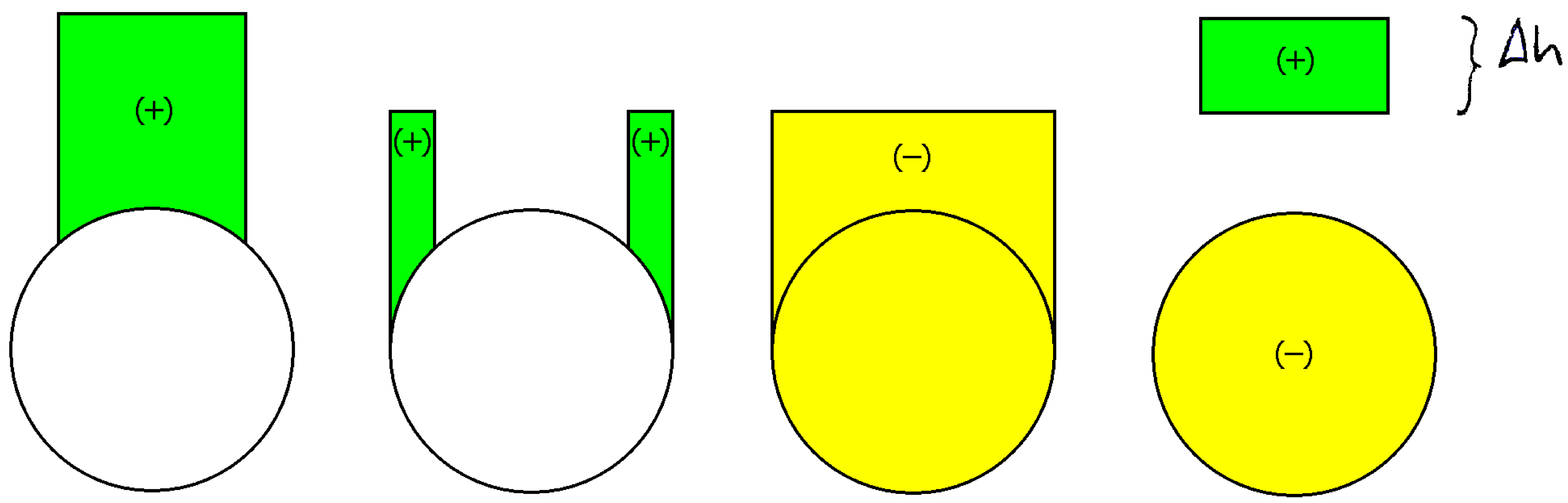
$$F_1 = \rho \cdot g \cdot \frac{h_1}{2} \cdot h_1 \cdot b = 9810\,N$$

$$F_2 = \rho g \cdot \frac{h_2}{2} \cdot h_2 \cdot b = 39240\,N$$

$$F = F_2 - F_1 = 29430\,N$$

(4-8)

(a)



$V_1 \qquad V_2 \qquad -V_3 \qquad V = V_1 + V_2 - V_3 = V_Z - V_K$

$$F_G = \rho_K \cdot g \cdot \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \rho_K \cdot g \cdot \frac{\pi}{6} D^3$$

$$F_A = \rho_W \cdot g\left(-V_K + V_Z\right) = \rho_W \cdot g\left(-\frac{\pi}{6}D^3 + \frac{\pi}{4}d^2 \cdot \Delta h\right)$$

$$\sum F = 0$$

$$\rho_K \cdot g \, \frac{\pi}{6} D^3 + \rho_W \, g\left(-\frac{\pi}{6}D^3 + \frac{\pi}{4}d^2 \Delta h\right) = 0$$

$$\rho_K \frac{\pi}{6} D^3 - \rho_W \frac{\pi}{6} D^3 + \rho_W \cdot \frac{\pi}{4} d^2 \Delta h = 0$$

$$\left(\rho_K - \rho_W\right) D^3 + \rho_W \cdot \frac{3}{2}\left(\frac{2}{3}D\right)^2 \cdot \Delta h = 0$$

$$\left(0{,}1\rho_W - \rho_W\right) D + \rho_W \cdot \frac{2}{3} \cdot \Delta h = 0$$

$$D = \frac{2}{3} \cdot \frac{\Delta h}{0{,}9}$$

$$D = 0{,}741 \, m$$

(b)

$$F_1 = \rho \cdot g \cdot \frac{h_1}{2} \cdot h_1 \cdot b = 39240 N$$

$$F_2 = \rho \cdot g \cdot \frac{h_1 - \Delta h}{2} \cdot (h_1 - \Delta h) \cdot b = 9810 N$$

$$F = F_1 - F_2 = 29430 N$$

(4-10)

$$F_1 = p \cdot A$$

$$= (p_0 + \rho_w \cdot g \cdot (h - \ell))\, \ell^2$$

$$= \left(10^5 \frac{N}{m^2} + 1000 \frac{kg}{m^3} \cdot 9{,}81 \frac{m}{s^2} (0{,}15 - 0{,}1)\, m\right) (0{,}1\, m)^2$$

$$= 1005\ N$$

$$F_2 = \rho_E \cdot g \cdot \ell^3$$

$$= 7860 \frac{kg}{m^3} \cdot 9{,}81 \frac{m}{s^2} \cdot (0{,}1\, m)^3$$

$$= 77\ N$$

$$F = F_1 + F_2 = 1082\ N$$

(4-11)

$$m = V_1 \cdot \rho_1 \quad \Rightarrow \quad V_1 = \frac{m}{\rho_1}$$

$$m \cdot g = \rho_2 \cdot g \cdot \left(V_1 + d^2 \frac{\pi}{4} \cdot h \right)$$

$$\rho_2 = \frac{m}{\frac{m}{\rho_1} + d^2 \frac{\pi}{4} \cdot h} = \frac{0{,}1\,kg}{\frac{0{,}1\,kg}{1280\,\frac{kg}{m^3}} + (0{,}006\,m)^2 \cdot \frac{\pi}{4} \cdot 0{,}028}$$

$$= \frac{0{,}01\,kg}{\frac{0{,}01\,kg}{1280\,\frac{kg}{m^3}} + (0{,}006\,m)^2 \cdot \frac{\pi}{4} \cdot 0{,}0285\,m}$$

$$= 1160\,\frac{kg}{m^3}$$

$$F_A = F_1 + F_2 = \rho_1 \cdot g \cdot V_1 + \rho_2 \cdot g \cdot V_2$$ Formel (4-16)

$$F_G = \rho_E \cdot g \cdot (V_1 + V_2)$$

$$F_A = F_G$$

$$\rho_W \, g \cdot A \cdot b + \rho_{Hg} \cdot g \cdot A \cdot a = \rho_E \cdot g \cdot A(a+b)$$

$$\rho_W \cdot b + \rho_{Hg} \cdot a = \rho_E \cdot (a+b)$$

$$a \cdot (\rho_{Hg} - \rho_E) = b\,(\rho_E - \rho_W)$$

$$\frac{a}{b} = \frac{\rho_E - \rho_W}{\rho_{Hg} - \rho_E}$$

$$= \frac{7850 - 1000}{13560 - 7850}$$

$$= 1{,}2$$

(4-13)

(a) $$F = F_H - F_N \cdot \mu_{Gl}$$

$$m \cdot a = m \cdot g \cdot \sin\alpha - mg \cdot \cos\alpha \, \mu_{Gl}$$

$$a = g(\sin\alpha - \mu_{Gl} \cdot \cos\alpha)$$

$$a = 9{,}81 \frac{m}{s^2}(\sin 20^\circ - 0{,}1 \cdot \cos 20^\circ)$$

$$= 2{,}43 \frac{m}{s^2}$$

$$\tan\beta = \frac{a \cdot \cos\alpha}{g \pm a \cdot \sin\alpha}$$ $\ominus$ a Formel (4-17)

$$\beta = \arctan\left(\frac{a \cdot \cos\alpha}{g - a \cdot \sin\alpha}\right)$$

$$= \arctan\left(\frac{2{,}43 \frac{m}{s^2} \cdot \cos 20^\circ}{9{,}81 \frac{m}{s^2} - 2{,}43 \frac{m}{s^2} \cdot \sin 20^\circ}\right)$$

$$= 14{,}3^\circ$$

$$V = b \cdot h \cdot l - b \cdot \Delta h \cdot l \cdot \frac{1}{2} \qquad \Delta h = l \cdot \tan(\alpha - \beta)$$

$$= b \cdot h \cdot l - b \cdot l^2 \cdot \tan(\alpha - \beta)$$

$$= 1m \cdot 1m \; 2m - 1m \cdot (2m)^2 \cdot \tan(20^\circ - 14{,}3^\circ)$$

$$= 2\,m^3 - 2\,m^3 \cdot \tan(5{,}7^\circ) \cdot \frac{1}{2}$$

$$= 1{,}9\,m^3$$

ω

(4-14)

$$p(z,r) = p_0 + \rho \cdot g \cdot z + \frac{1}{2}\rho r^2 \omega^2$$ (a)

$$p_G = p_0 + \rho \cdot g \cdot H + \frac{1}{2}\rho \cdot R^2 \cdot \omega^2$$

$$p_B = p_0 + \rho \cdot g \cdot (H + R)$$

$$p_C = p_B$$

$$p_0 + \rho \cdot g \cdot H + \frac{1}{2}\rho R^2 \omega^2 = p_0 + \rho \cdot g \cdot (H+R)$$

$$\frac{1}{2}\rho R^2 \omega^2 = g \cdot R$$

$$\omega = \sqrt{\frac{2 \cdot g}{R}}$$

$$= \sqrt{\frac{2 \cdot 9{,}81\, m}{19{,}5 \cdot cm\ s^2}}$$

$$= 10{,}03 \cdot \frac{1}{s}$$

$$p(\beta) = p_0 + \rho \cdot g(H + R \cdot \cos\beta) + \frac{1}{2}\rho \cdot (R \cdot \sin\beta)^2 \cdot \omega^2$$ (b)

$$\frac{\cancel{p(\beta)}}{d\beta} = -\rho \cdot g \cdot R \cdot \sin\beta + \rho \cdot R^2 \cdot \omega^2 \cdot \sin\beta \cdot \cos\beta = 0 \quad | : \sin\beta \neq 0$$

$$-\rho \cdot g \cdot R + \rho \cdot R^2 \cdot \frac{2 \cdot g}{R} \cdot \cos\beta = 0 \quad | : \rho \cdot g \cdot R$$

$$-1 + \beta\, 2 \cdot \cos\beta = 0$$

$$\beta = \arccos\left(\frac{1}{2}\right)$$

$$= 60^\circ$$

(7-1)

(a) $\dot{V}_2 = 2 \cdot \dot{V}_3$

$$c_2 d_2^2 = 2 c_3 d_3^2$$

$$c_1 \cdot A_1 = c_2 \cdot A_2 + c_3 \cdot A_3 \qquad V_1 = 42 \frac{m^3}{h}$$

$$c_1 \cdot d_1^2 = c_2 \cdot d_2^2 + c_3 \cdot d_3^2 \qquad d_1 = d_2 = 0{,}1\,m$$

$$c_3 \cdot d_1^2 = 2 c_3 d_3^2 + c_3 \cdot d_3^2 \qquad c_1 = c_3$$

$$d_1^2 = 3 d_3^2$$

$$d_3 = \frac{d_1}{\sqrt{3}} = \frac{100\,mm}{\sqrt{3}} = 58\,mm$$

$$c_1 = \frac{\dot{V}_1}{d_1^2} \cdot \frac{4}{\pi} = \frac{\frac{42\,m^3}{3600\,s} \cdot 4}{(0{,}1\,m)^2 \cdot \pi} = 1{,}485 \frac{m}{s}$$

$$c_2 = \frac{2 c_3 d_3^2}{d_2^2} = \frac{2 c_1}{d_1^2} \frac{d_1^2}{3} = \frac{2}{3} c_1 = 0{,}99 \frac{m}{s}$$

(7-2)

$$c_1 \cdot d_1^2 = c_0 \cdot d_0^2 + c_2 \, d_2^2$$

$$c_0 = \frac{c_1 \cdot d_1^2 - c_2 \, d_2^2}{d_0^2}$$

$$= \frac{10 \, \frac{m}{s} \cdot (0{,}01 \, m)^2 - 5 \, \frac{m}{s} \cdot (0{,}012 \, m)^2}{(0{,}2 \, m)^2}$$

$$= 7 \cdot 10^{-3} \, \frac{m}{s}$$

$$= 0{,}7 \, \frac{cm}{s}$$

(8-1)

$$p_1 = p_0 + \rho_w \cdot g \cdot h_1$$

$$p_2 = p_0 + \rho_w \cdot g \cdot h_2$$

$$p_1 - p_2 = \rho_w \cdot g (h_1 - h_2)$$

Konti: $c_1 \cdot A_1 = c_2 \cdot A_2$

$$c_2 = c_1 \cdot \frac{A_1}{A_2} = c_1 \left(\frac{d_1}{d_2}\right)^2$$

$$c_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V} \cdot 4}{d_1^2 \pi} = \frac{10^5 m^3 4}{3600 s \cdot (1{,}2 m) \cdot \pi} = 24{,}56 \frac{m}{s}$$

Energie: $\frac{\rho_L}{2} \cdot c_1^2 + p_1 = \frac{\rho_L}{2} \cdot c_2^2 + p_2$

$$\frac{\rho_L}{2} \cdot c_1^2 + p_1 = \frac{\rho_L}{2} c_1^2 \left(\frac{d_1}{d_2}\right)^4 + p_2$$

$$\left(\frac{d_1}{d_2}\right)^4 = \frac{p_1 - p_2}{c_1^2 \cdot \frac{\rho_L}{2}} + 1$$

$$= \frac{\rho_w \cdot g \cdot \Delta h \cdot 2}{c_1^2 \cdot \rho_L} + 1$$

$$= \frac{10^3 \frac{kg}{m^3} \cdot 9{,}81 \frac{m}{s^2} \cdot 0{,}1 m \cdot 2}{(24{,}56 \frac{m}{s})^2 \cdot 1{,}2 \frac{kg}{m^3}} + 1$$

$$= 3{,}71$$

$$\Rightarrow \quad \frac{d_1}{d_2} = \sqrt[4]{3{,}71} = 1{,}388$$

(8-2) Energiesatz: $c_1 = 0 \qquad p_1 = p_x = p_0$

$$\frac{1}{2} c_1^2 + \frac{p_1}{\rho} + g \cdot h = \frac{1}{2} \cdot c_x^2 + \frac{p_x}{\rho} + g \cdot (-x)$$

$$g \cdot h = \frac{1}{2} c_x^2 + g \cdot (-x)$$

$$c_x = \sqrt{2g(h+x)}$$

Kontigleichung:

$$A \cdot c_x = A_0 \cdot c_0 \qquad c_0 = \sqrt{2gh}$$

$$A \sqrt{2g(h+x)} = A_0 \sqrt{2gh}$$

$$A = A_0 \sqrt{\frac{h}{h+x}}$$

(8-3)

$$\frac{1}{2} c_2^2 + g \cdot z_2 + \frac{p_2}{\rho} = \frac{1}{2} c_3^2 + g \cdot z_3 + \frac{p_3}{\rho}$$

(a)

$p_2 = p_{min}$ $\quad p_3 = p_0$ $\quad z_2 = 0$ $\quad c_3 = c_2$ $\quad z_3 = -l$

$$\frac{p_{min}}{\rho} = g \cdot (-l) + \frac{p_0}{\rho}$$

$$l = \frac{p_0 - p_{min}}{\rho \cdot g}$$

$$= \frac{(1 - 0{,}2) \cdot 10^5}{1000 \frac{kg}{m^3} \cdot 9{,}81 \frac{m}{s^2}} \frac{N}{m^2}$$

$$= 8{,}155 \, m$$

(b)

$$\dot{m} = \rho \cdot \dot{V}_3 = \rho \cdot c_3 \cdot A_3 = \rho \cdot c_3 \cdot d_3^2 \frac{\pi}{4} \qquad c_3 = \sqrt{2g(h+l)}$$

$$d_3 = \sqrt{\frac{\dot{m} \cdot 4}{\rho \cdot c_3 \cdot \pi}} = \sqrt{\frac{\dot{m} \cdot 4}{\rho \cdot \pi \cdot \sqrt{2g(h+l)}}}$$

$$= \sqrt{\frac{5 \, kg \cdot 4}{s \cdot \pi \cdot 1000 \frac{kg}{m^3} \cdot \sqrt{2 \cdot 9{,}81 \frac{m}{s^2} (2 + 8{,}155) m}}}$$

$$= 2{,}1 \, cm$$

(8-4)

(a)

$$\frac{1}{2}c_3^2 + g\cdot a + \frac{p_3}{\rho} = \frac{1}{2}c_2^2 + g\cdot h_0 + \frac{p_0}{\rho} \qquad c_2^2 = 2gH$$

$$\frac{1}{2}c_3^2 + g\cdot a + \frac{p_{min}}{\rho} = gH + \frac{p_0}{\rho} \qquad p_3 = p_{min}$$

$$c_3^2 = 2gH + 2\frac{p_0}{\rho} - 2ga - 2\frac{p_{min}}{\rho}$$

$$c_3 = 15{,}76\ \frac{m}{s}$$

$$c_2\cdot d_2^2 = c_3\cdot d_3^2$$

$$d_2 = d_3\cdot\sqrt{\frac{c_3}{c_2}} = d_3\sqrt{\frac{c_3}{\sqrt{2gH}}} \qquad d_2 = D$$

$$D = 0{,}1\,m\cdot\sqrt{15{,}76\,\frac{m}{s}\cdot\frac{1}{\sqrt{2\cdot 9{,}81\frac{m}{s^2}\cdot 10\,m}}} = 10{,}6\,cm$$

(b)

$$\dot{V}_2 = c_2\cdot A_2 = \sqrt{2gH}\cdot d^2\frac{\pi}{4}$$

$$= \sqrt{2\cdot 9{,}81\frac{m}{s^2}\cdot 10\,m}\cdot(0{,}106\,m)^2\cdot\frac{\pi}{4} = 0{,}1238\ \frac{m^3}{s}$$

(8-5)

$$c_1 = \frac{\dot{V}}{A_1} = \frac{V}{t}\cdot\frac{4}{D^2\cdot\pi} = \frac{5\cdot 10^{-6}\,m^3\cdot 4}{10\,s\;\;0{,}01^2\,m^2\cdot\pi}$$

$$= 6{,}37\cdot 10^{-3}\,\frac{m}{s}$$

$$c_2 = \frac{V\cdot 4}{t\cdot d^2\,\pi} = \frac{5\cdot 10^{-6}\,m^3\cdot 4}{10\,s\cdot 0{,}4^2\cdot 10^{-6}\,m^2\cdot\pi} = 3{,}99\,\frac{m}{s}$$

$$\Delta p_v = \varphi_{12}\cdot\rho = \frac{1}{2}\cdot c_2^2\cdot\zeta\cdot\rho = \frac{1}{2}\,c_2^2\cdot\lambda\cdot\frac{l}{d}\cdot\rho$$

$$= \frac{1}{2}\,c_2^2\cdot\frac{64}{Re}\cdot\frac{l}{d}\cdot\rho = \frac{1}{2}\,c_2^2\cdot\frac{64\cdot\nu\cdot l}{c_2\cdot d\cdot d}\cdot\rho$$

$$= c_2\cdot 32\cdot\nu\cdot l\cdot\rho\cdot\frac{1}{d^2}$$

$$= 3{,}99\,\frac{m}{s}\cdot 32\cdot 5\cdot 10^{-6}\,\frac{m^2}{s}\cdot 6\cdot 10^{-2}\,m\cdot 980\,\frac{kg}{m^3}\cdot\frac{1}{(0{,}4\cdot 10^{-3})^2}$$

$$= 2{,}34\text{ bar}$$

$$\frac{1}{2}c_1^2 + \frac{p_1}{\rho} = \frac{1}{2}c_2^2 + \frac{p_2}{\rho} + \frac{\Delta p_v}{\rho} \qquad p_2 = p_0$$

$$p_1 = \frac{\rho}{2}(c_2^2 - c_1^2) + p_0 + \Delta p_v$$

$$= \frac{920}{2}\,\frac{kg}{m^3}\,(3{,}99^2 - (6{,}37\cdot 10^{-3})^2)\,\frac{m^2}{s^2} + (1 + 2{,}34)\,bar$$

$$= 3{,}43\text{ bar}$$

$$F = p\cdot A_1 = (p_1 - p_0)A_1 = (p_1 - p_0)\,D^2\,\frac{\pi}{4}$$

$$= (3{,}43 - 1)\cdot 10^5\,\frac{N}{m^2}\cdot 0{,}01^2\,m^2\cdot\frac{\pi}{4}$$

$$= 19{,}05\,N$$

(8.6)

(a) $$c = \frac{\dot{V}}{A} = \frac{\Delta V}{\Delta t} \cdot \frac{4}{d^2 \pi} = \frac{100 \cdot 10^{-3} m^3 \cdot 4}{12{,}73 s \cdot 0{,}05^2 m^2 \pi} = 4 \frac{m}{s}$$

$$\varphi_{12} = \frac{1}{2} c^2 \cdot (\zeta + \zeta_E) = \frac{1}{2} c^2 \left(\lambda \frac{l}{d} + \zeta_E\right)$$

$$\frac{1}{2} c_1^2 + g h_1 + \frac{p_1}{\rho} = \frac{1}{2} c_2^2 + g h_2 + \frac{p_2}{\rho} + \varphi_{12}$$

$c_1 = 0$ $h_1 = h$ $p_1 = p_0$ $c_2 = c$ $h_2 = 0$ $p_2 = p_0$

$$g \cdot h = \frac{1}{2} c^2 + \varphi_{12}$$

$$g \cdot h - \frac{1}{2} c^2 = \frac{1}{2} c^2 \left(\lambda \cdot \frac{l}{d} + \zeta_E\right)$$

$$2 g h \cdot \frac{1}{c^2} - 1 = \lambda \cdot \frac{l}{d} + \zeta_E$$

$$\lambda = \frac{d}{l} \left(2 g h \cdot \frac{1}{c^2} - 1 - \zeta_E\right)$$

$$= \frac{0{,}05 m}{16{,}78 m} \left(2 \cdot 9{,}81 \frac{m}{s^2} \cdot 8{,}28 m \cdot \frac{1}{16} \frac{s^2}{m^2} - 1 - 1\right)$$

$$= 0{,}0243$$

(b) $$\lambda = 0{,}0243 \xrightarrow{\text{Tabelle}} \frac{k_s}{d} = 2 \cdot 10^{-3}$$

$$R_e = \frac{c \cdot d}{\nu} = 4 \frac{m}{s} \cdot \frac{0{,}05 m}{10^{-6} m^2/s} = 2 \cdot 10^5 \rightarrow \text{turbulent}$$

$$k_s = 2 \cdot 10^{-3} \cdot 0{,}05 m = 0{,}1 mm$$

(8-7)

(a)

$$\frac{p_A}{\rho} = \frac{p_B}{\rho} + \varphi_{12}$$

$$\varphi_{12} = \frac{p_A \cdot p_B}{\rho} = 105{,}66\ \frac{m^2}{s^2}$$

$$\varphi_{34} = \frac{p_C - p_D}{\rho} = 85{,}8\ \frac{m^2}{s^2}$$

$$\varphi_{12} = \frac{1}{2} c^2 \zeta$$

$$\varphi_{12} = \frac{1}{2} c^2 \cdot \lambda \cdot \frac{l}{d}$$

$$\varphi_{12} = \frac{1}{2} c^2 \cdot \frac{0{,}316}{\sqrt[4]{Re}} \cdot \frac{l}{d}$$

$$\varphi_{12} = \frac{1}{2} c^2 \frac{0{,}316}{\sqrt[4]{\frac{c \cdot d}{\nu}}} \frac{l}{d}$$

$$\left(\frac{2}{0{,}316} \varphi_{12} \frac{d}{l} \sqrt[4]{\frac{d}{\nu}}\right)^{\frac{4}{7}} = c$$

$$c_1 = \left(\frac{2}{0{,}316} \cdot 105{,}66\ \frac{m^2}{s^2} \cdot \frac{0{,}05\,m}{500\ m} \sqrt[4]{0{,}05\ \frac{m \cdot s}{10^{-6}\,m^2}}\right)^{\frac{4}{7}} = 1\ \frac{m}{s}$$

$$c_2 = \left(\frac{2}{0{,}316} \cdot 85{,}8\ \frac{m^2}{s^2} \cdot \frac{0{,}05\,m}{600\ m} \sqrt[4]{\frac{0{,}05\ m \cdot s}{10^{-6}\,m^2}}\right)^{\frac{4}{7}} = 0{,}79998\ \frac{m}{s}$$

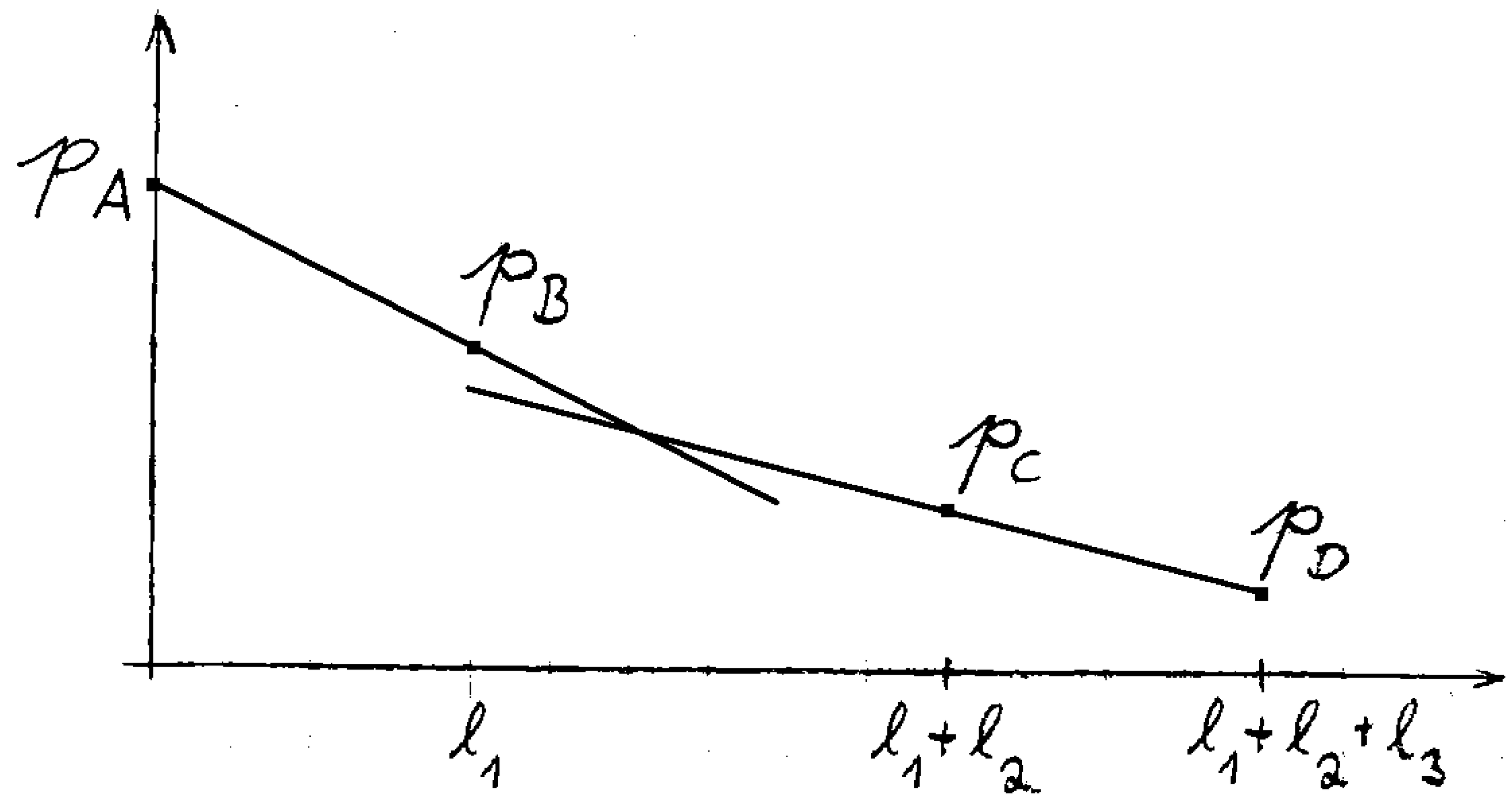
$$\dot{V}_L = A \cdot (c_1 - c_2) = d^2 \frac{\pi}{4} (c_1 - c_2)$$

$$= (0{,}05\,m)^2 \cdot \frac{\pi}{4} (1 - 0{,}79998)\ \frac{m}{s} = 3{,}927 \cdot 10^{-4}\ \frac{m^3}{s}$$

$$= 0{,}3927\ \frac{l}{s}$$

(8-7)

(c)



$$p_A - \frac{p_A - p_B}{l_1} \cdot x = p_C - \frac{p_C - p_D}{l_3}\left(x - (l_1 + l_2)\right)$$

$$\left(\frac{p_C - p_D}{l_3} - \frac{p_A - p_B}{l_1}\right) \cdot x = p_C - p_A + (l_1 + l_2)\,\frac{p_C - p_D}{l_3}$$

$$x = \frac{p_C - p_A + (l_1 + l_2)\,\frac{p_C - p_D}{l_3}}{\frac{p_C - p_D}{l_3} - \frac{p_A - p_B}{l_1}}$$

$$= \frac{l_3\,\frac{p_C - p_A}{p_C - p_D} + l_1 + l_2}{1 - \frac{l_3}{l_1}\,\frac{p_A - p_B}{p_C - p_D}}$$

$$= \frac{600\,m \cdot \frac{5{,}2 - 10}{5{,}2 - 4{,}342} + 500\,m + 2000\,m}{1 - \frac{600}{500} \cdot \frac{10 - 8{,}9434}{5{,}2 - 4{,}342}}$$

$$= 1793\,m$$

(8-8) $\varphi_{12} = \frac{1}{2} c^2 \cdot \zeta \qquad P_R = \varphi_{12} \dot{m} \qquad Re = \frac{c \cdot d}{\nu} \qquad \zeta = \lambda \cdot \frac{\ell}{d}$

$$c = \frac{\dot{V}}{A} = \frac{\dot{V}\,4}{d^2 \pi} = \frac{6{,}283\, m^3 \cdot 4}{s \cdot 4 m^2 \, \pi} = 2 \frac{m}{s}$$

$$Re = \frac{c \cdot d}{\nu} = \frac{2 m \cdot 2 m \cdot 1 \cdot s}{s \quad 10^{-6} m^2} = 4 \cdot 10^6$$

$$\left.\begin{aligned} \frac{k_{sn}}{d} &= \frac{0{,}004\, m}{2\, m} = 0{,}001 = 1 \cdot 10^{-3} \\ \frac{k_{sa}}{d} &= \frac{0{,}02\, m}{2\, m} = 0{,}01 = 1 \cdot 10^{-2} \end{aligned}\right\} \text{Tabelle}$$

$$\Rightarrow \Delta\lambda = 14{,}5 \cdot 10^{-3}$$

$$\dot{m} = \rho \cdot \dot{V} = 1000 \frac{kg}{m^3} \cdot 6{,}283 \frac{m^3}{s} = 6283 \frac{kg}{s}$$

$$P_R = \varphi_{12} \cdot \dot{m} = \frac{1}{2} c^2 \cdot \zeta \cdot \dot{m} = \frac{1}{2} c^2 \lambda \cdot \frac{\ell}{d} \cdot \dot{m}$$

$$= \frac{1}{2}\left(2 \frac{m}{s}\right)^2 \cdot 14{,}5 \cdot 10^{-3} \cdot \frac{1000\, m}{2\, m} \cdot 6283 \frac{kg}{s}$$

$$= 14{,}5 \cdot 6283 \frac{kg\, m}{s^2} \frac{m}{s}$$

$$= 91103{,}5\, W \qquad P_{el} = P_R \frac{1}{\eta_e} =$$

$$P_{el} = \frac{P_R}{\eta_e} = \frac{91103{,}5\, W}{0{,}8} = 113\,875{,}375\, W$$

Ersparnis: $BK_a - BK_n = P_{el} \cdot 365 \cdot 24 h \cdot \frac{0{,}1\, €}{kWh}$

ΔBK $= 113\,875{,}375\, W \cdot 365 \cdot 24 h \cdot \frac{0{,}1\, €}{kWh}$

$= 99752\, €$

(8-9) $\dot m_1 = \dot m_2 + \dot m_3$

$$c_1 \cdot A_1 = c_2 A_2 + c_3 A_3$$

$$c_1 \cdot D^2 = c_2 D^2 + c_3 d^2$$

$$c_1 = c_2 + c_3 \frac{d^2}{D^2}$$

$$= c_2 + c_3 \left(\frac{4}{10}\right)^2$$

$$= c_2 + c_3 \frac{4}{25}$$

$$\dot V = \frac{\pi \cdot d^4}{128 \cdot \eta \cdot l}(p_1 - p_2) \qquad (8-50)$$

$$c = \frac{\dot V}{A} = \frac{\dot V\, 4}{d^2 \pi} = \frac{\pi d^4 \cdot}{128 \eta l}(p_1 - p_2) \cdot \frac{4}{d^2 \pi}$$

$$= \frac{d^2}{32 \eta l}(p_1 - p_2) = \frac{d^2}{32 \cdot \nu \cdot \rho \cdot l}(p_1 - p_2)$$

$$c_1 = \frac{(0{,}01\,m)^2 (100 - 98{,}75) \cdot 10^5\, N}{32 \cdot 25 \cdot 10^{-6} \frac{m^2}{s} \cdot 1000 \frac{kg}{m^3} \cdot m^2 \cdot 6{,}25\, m} = 2{,}5 \frac{m}{s}$$

$$c_2 = \frac{(0{,}01) m^2 (98{,}75 - 97{,}15) \cdot 10^5 N}{32 \cdot 25 \cdot 10^{-6} \frac{m^2}{s} \cdot 1000 \frac{kg}{m^3} \cdot m^2 \cdot 10\, m} = 2 \frac{m}{s}$$

$$c_3 = (c_1 - c_2) \cdot \frac{25}{4} = (2{,}5 - 2) \frac{m}{s} \cdot \frac{25}{4} = 3{,}125 \frac{m}{s}$$

$$c_3 = \frac{d^2}{32 \cdot \nu \cdot \rho \cdot l}(p_2 - p_4)$$

$$p_4 = p_2 - c_3 \cdot 32 \cdot \nu \cdot \rho \cdot l \cdot \frac{1}{d^2}$$

$$= 98{,}75 \cdot 10^5 \frac{N}{m^2} - 3{,}125 \frac{m}{s} \cdot 32 \cdot 25 \cdot 10^{-6} \frac{m^2}{s} \cdot 1000 \frac{kg}{m^3} \cdot 1m \cdot \frac{1}{(0{,}004m)^2}$$

$$= 97{,}1875 \text{ bar}$$

$$Re_1 = \frac{c_1 d}{\nu} = \frac{2{,}5 \frac{m}{s} \cdot 10^{-2} m \cdot s}{25 \cdot 10^{-6} m^2} = 1000$$

$$Re_3 = \frac{c_3 \cdot d}{\nu} = \frac{3{,}125 m \cdot 4 \cdot 10^{-3} m \cdot s}{25 \cdot 10^{-6} m^2} = 500$$

(8-10)

(a)

$$\lambda_a = \left(\frac{1}{1{,}74 - 2\cdot\log\left(\frac{2\cdot k_s}{d}\right)}\right)^2 = \left(\frac{1}{1{,}74 - 2\cdot\log\left(2\cdot\frac{0{,}5\cdot 10^{-3}\,m}{0{,}1\,m}\right)}\right)^2 = 0{,}0304$$

$$g\cdot H = \frac{1}{2}c^2\left(1 + \frac{\lambda_a\cdot l}{d}\right)$$

$$c_a = \sqrt{\frac{2\cdot g\cdot H}{1+\frac{\lambda_a\cdot l}{d}}} = \sqrt{\frac{2\cdot 9{,}81\frac{m}{s^2}\cdot 30\,m}{1+\frac{0{,}0304\cdot 20\,m}{0{,}1}}} = 9{,}12\ \frac{m}{s}$$

$$\dot V_a = c\cdot A = 9{,}12\,\frac{m}{s}\cdot 78{,}54\cdot 10^{-4}\,m^2 = 258\ \frac{m^3}{h}$$

(b)

$$d_h = \frac{4A}{U} = \frac{4\cdot A}{4\sqrt{A}} = \sqrt{A} = \sqrt{78{,}54\cdot 10^{-4}\,m^2} = 88{,}6\cdot 10^{-3}\,m$$

$$\lambda_b = \left(\frac{1}{1{,}74 - 2\cdot\log\left(\frac{2\cdot k_s}{d_h}\right)}\right) = \left(\frac{1}{1{,}74 - 2\cdot\log\left(\frac{2\cdot 0{,}5\cdot 10^{-3}\,m}{88{,}6\cdot 10^{-3}\,m}\right)}\right)^2 = 0{,}0315$$

$$c_b = \sqrt{\frac{2\cdot g\cdot H}{1+\frac{\lambda_b\cdot l}{d_h}}} = \sqrt{\frac{2\cdot 9{,}81\,m/s^2\cdot 30\,m}{1+\frac{0{,}0315\cdot 20\,m}{88{,}6\cdot 10^{-3}\,m}}} = 8{,}52\ \frac{m}{s}$$

$$\dot V_b = c_b\cdot A = 8{,}52\,\frac{m}{s}\cdot 78{,}54\cdot 10^{-4}\,m^2 = 241\ \frac{m^3}{h}$$

(8-11)

(a) Fall a:
$$\frac{1}{\sqrt{\lambda}} = 1{,}74 - 2\log\left(\frac{2k_s}{d}\right)$$

$$\lambda = \left(1{,}74 - 2\log\left(2\cdot\frac{0{,}001}{0{,}1}\right)\right)^{-2} = \boxed{37{,}88\cdot 10^{-3}}$$

$$\zeta = \frac{\lambda\cdot l}{d} = \frac{37{,}88\cdot 10^{-3}\cdot 10m}{0{,}1m} = \boxed{3{,}788}$$

$$g\cdot H = \frac{1}{2}c_a^2 + \frac{1}{2}c_a^2\cdot\zeta$$

$$c_a = \sqrt{\frac{2gH}{1+\zeta}} = \sqrt{\frac{2\cdot 9{,}81\frac{m}{s^2}\cdot 20m}{1+3{,}788}} = \boxed{9{,}053\,\frac{m}{s}}$$

Fall b:
$$\zeta_D = 0{,}1\left(1-\left(\frac{d}{D}\right)^4\right) = 0{,}1\left(1-\left(\frac{0{,}1}{0{,}2}\right)^4\right) = \boxed{93{,}75\cdot 10^{-3}}$$

$$c_e = c_b\frac{D^2}{d^2} = c_b\left(\frac{D}{d}\right)^2 = c_b\cdot\left(\frac{0{,}2}{0{,}1}\right)^2 = c_b\cdot 4$$

$$g\cdot H = \frac{1}{2}c_b^2 + \frac{1}{2}c_b^2(\zeta+\zeta_D)$$

$$= \frac{1}{2}c_b^2 + \frac{1}{2}c_b^2\left(\frac{D}{d}\right)^4(\zeta+\zeta_D)$$

$$= \frac{1}{2}c_b^2\left(1+2^4(\zeta+\zeta_D)\right)$$

$$c_b = \sqrt{2\cdot g\cdot H\left(1+16\,(\zeta+\zeta_D)\right)^{-1}}$$

$$= \sqrt{\frac{2\cdot 9{,}81\frac{m}{s^2}\cdot 20m}{1+16(3{,}788+93{,}75\cdot 10^{-3})}} = \boxed{2{,}494\,\frac{m}{s}}$$

$$c_e = 4\cdot c_b = \boxed{9{,}974\,\frac{m}{s}}$$

$$\Delta\dot{m} = \dot{m}_b - \dot{m}_a = \rho\cdot(\dot{V}_b - \dot{V}_a) = \rho(c_b\cdot D^2 - c_a d^2)\frac{\pi}{4}$$

$$= \rho(c_e d^2 - c_a d^2)\frac{\pi}{4} = \rho(c_e - c_a)d^2\frac{\pi}{4}$$

$$= 1000\frac{kg}{m^3}(9{,}974 - 9{,}053)\frac{m}{s}\cdot(0{,}1m)^2\frac{\pi}{4}$$

$$= \boxed{7{,}2\,\frac{kg}{s}}$$

(b) $P_{Ra} = \varphi_{12a} \cdot \dot{m}_a = \frac{1}{2} c_a^2 \cdot \zeta \cdot \rho \cdot c_a \cdot d^2 \frac{\pi}{4} = c_a^3 \cdot \zeta \cdot \rho \cdot d^2 \frac{\pi}{8}$

$$= \left(9{,}053 \frac{m}{s}\right)^3 \cdot 3{,}788 \cdot 1000 \frac{kg}{m^3} \cdot (0{,}1 m)^2 \cdot \frac{\pi}{8}$$

$$= \boxed{11{,}036 \text{ kW}}$$

$P_{Rb} = \varphi_{12b} \cdot \dot{m}_b = \frac{1}{2} c_e^2 (\zeta + \zeta_D) \cdot \rho \cdot c_e \cdot d^2 \frac{\pi}{4}$

$$= c_e^3 (\zeta + \zeta_D) \cdot \rho \, d^2 \frac{\pi}{8}$$

$$= \left(9{,}074 \frac{m}{s}\right)^3 \left(3{,}788 + 93{,}75 \cdot 10^{-3}\right) \cdot 1000 \frac{kg}{m^3} (0{,}1 m)^2 \frac{\pi}{8}$$

$$= \boxed{15{,}126 \text{ kW}}$$

$\Delta P_R = P_{Rb} - P_{Ra} = (15{,}126 - 11{,}036) \text{ kW} = 4{,}1 \text{ kW}$

(8-12)

(a) $$d_h = 2\frac{a\cdot b}{a+b} = 2\frac{4b\cdot b}{4b+b} = 2\frac{4b^2}{5b} = \frac{8}{5}b$$

$$b = \frac{5}{8}d_h = \frac{5}{8}\cdot 002\,m = 12{,}5\cdot 10^{-3}\,m$$

$$A = a\cdot b = 4b\cdot b = 4b^2 = 4(12{,}5\cdot 10^{-3}\,m)^2 = 6{,}25\cdot 10^{-4}\,m^2$$

$$c = \frac{\dot V}{A} = \frac{6{,}25\,10^{-3}\,m^3/s}{6{,}25\cdot 10^{-4}\,m^2} = 10\,\frac{m}{s}$$

$$Re = \frac{c\cdot d_h}{\nu} = 10\frac{m}{s}\,\frac{0{,}02\,m}{10^{-6}\,\frac{m^2}{s^2}} = 2\cdot 10^5$$

$$\frac{k_s}{d_h} = \frac{0{,}01\ 10^{-3}\,m}{0{,}02\ m} = 5\cdot 10^{-4} \quad \Rightarrow \lambda = 0{,}0192$$

$$\Delta p = \frac{1}{2}c^2\zeta\rho$$

$$= \frac{1}{2}c^2\left(\frac{\lambda\cdot l}{d_h} + \zeta_A\right)\rho$$

$$= \frac{1}{2}\left(10\frac{m}{s}\right)^2\left(\frac{0{,}0192\cdot 30\,m}{0{,}02\,m} + 1\right)1000\,\frac{kg}{m^3}$$

$$= 14{,}9\ bar$$

$$p_1 = \Delta p + \rho\cdot g\cdot h = 14{,}9\,bar + 1000\,\frac{kg}{m^3}\cdot 9{,}81\,\frac{m}{s^2}\cdot 10\,m = 15{,}881\,bar$$

(b) $$P_R = \dot m\cdot \varphi_{12} = \rho\cdot\dot V\cdot\frac{1}{2}c^2\left(\frac{\lambda\cdot l}{d} + \zeta_A\right)$$

$$= 1000\,\frac{kg}{m^3}\cdot 6{,}25\cdot 10^{-3}\,\frac{m^3}{s}\cdot\frac{1}{2}\left(10\frac{m}{s}\right)^2\left(\frac{0{,}0192\cdot 30\,m}{0{,}02} + 1\right)$$

$$= 9{,}3125\ kW$$

mit $\lambda = 0{,}0188$ kommt $P_R = 9{,}125\ kW$ raus

(8-13)

$$\frac{1}{2}c_1^2 + g\cdot z_1 + \frac{p_1}{\rho} = \frac{1}{2}c_2^2 + g z_2 + \frac{p_2}{\rho} + \varphi_{12}$$

$$c_1 = c_2 \quad z_1 - z_2 = H \quad p_1 = p_2$$

$$g\cdot H = \varphi_{12}$$

$$\varphi_{12} = 9{,}81\,\frac{m}{s^2}\cdot 10\,m = 98{,}1\,\frac{m^2}{s^2}$$

$$\varphi_{12} = \frac{1}{2}c_1^2\left(\frac{\lambda\cdot l_1}{d_1} + \zeta_1\right) + \frac{1}{2}c_2^2\left(\frac{\lambda\, l_2}{d_2} + \zeta_2 + \zeta_3\right)$$

$$\dot{V} = A_1 c_1 = A_2 c_2 = d_1^2\frac{\pi}{4}\cdot c_1 = d_2^2\frac{\pi}{4}c_2$$

$$\Rightarrow \quad c_2 = c_1\cdot\left(\frac{d_1}{d_2}\right)^2 \qquad \zeta_2 = \left(\left(\frac{d_2}{d_1}\right)^2 - 1\right)^2 = \left(\left(\frac{0{,}3}{0{,}2}\right)^2 - 1\right)^2 = 1{,}5625$$

$$\varphi_{12} = \frac{1}{2}c_1^2\left(\frac{\lambda\, l_1}{d_1} + \zeta_1\right) + \frac{1}{2}c_1^2\left(\frac{d_1}{d_2}\right)^4\left(\frac{\lambda\, l_2}{d_2} + \zeta_2 + \zeta_3\right)$$

$$c_1 = \sqrt{\frac{\varphi_{12}}{\frac{1}{2}\left(\lambda\cdot\frac{l_1}{d_1} + \zeta_1\right) + \frac{1}{2}\left(\frac{d_1}{d_2}\right)^4\left(\frac{\lambda\cdot l_2}{d_2} + \zeta_2 + \zeta_3\right)}}$$

$$c_1 = \sqrt{\frac{98{,}1\,\frac{m}{s^2}}{\frac{1}{2}\left(\frac{0{,}03\cdot 20}{0{,}2} + 0{,}6\right) + \frac{1}{2}\left(\frac{0{,}2}{0{,}3}\right)^4\left(\frac{0{,}03\cdot 20}{0{,}3} + 1{,}5625 + 1\right)}}$$

$$= 5{,}72\,\frac{m}{s}$$

$$\dot{V} = c_1\cdot A_1 = c_1\cdot d_1^2\,\frac{\pi}{4} = 5{,}72\,\frac{m}{s}\cdot(0{,}2\,m)^2\cdot\frac{\pi}{4}$$

$$= 0{,}17965\cdot\frac{m^3}{s} = 179{,}65\,\frac{l}{s}$$