Klausur vom <u>28.09.04</u> Aufgabe 1

$$2z^3 - 6z^2 + 18z + 26 = 0$$

Nullstelle durch raten $z_1 = -1$

	2	-6	18	26
	–	-2	8	-26
$z=-1$	2	-8	26	0

$\Rightarrow \quad 2z^2 - 8z + 26 = 0$

$$z^2 - 4z + 13 = 0$$

$$z_{3/4} = 2 \pm \sqrt{4-13} = 2 \pm \sqrt{-9} = 2 \pm 3j$$

$z_1 = -1$
$z_2 = 2+3j$
$z_3 = 2-3j$

Klausur <u>11.02.04</u> 1

ges.: I $z \cdot \bar{z} = 5$

II $\dfrac{z}{\bar{z}} = \dfrac{3+4j}{5}$

ges.: $z = ?$

I $\curvearrowright \bar{z} = \dfrac{5}{z}$

I in II $\quad \dfrac{z \cdot z}{5} = \dfrac{3+4j}{5} \quad \curvearrowright \quad z^2 = 3+4j$

$$z = \sqrt{3+4j}$$

$$z_k = \sqrt[q]{r}\left[\cos\frac{\varphi + k\cdot 2\pi}{q} + j\sin\frac{\varphi + k\cdot 2\pi}{q}\right]$$

$q = 2$
$r = \sqrt{3^2+4^2} = 5$
$\varphi = \arctan\left(\frac{4}{3}\right)$

$k=0$: (Hauptwert) $\quad z_1 = \sqrt[2]{5}\left[\cos\left(\dfrac{\arctan\left(\frac{4}{3}\right)}{2}\right) + j\sin\left(\dfrac{\arctan\left(\frac{4}{3}\right)}{2}\right)\right] = \underline{\underline{2+j}}$

$k=1$: (Nebenwert) $\quad z_2 = \sqrt[2]{5}\left[\cos\left(\dfrac{\arctan\left(\frac{4}{3}\right)+2\pi}{2}\right) + j\sin\left(\dfrac{\arctan\left(\frac{4}{3}\right)+2\pi}{2}\right)\right] = \underline{\underline{-2-j}}$

Klausur <u>12.02.03</u> (A1)

ges.: $x^3(1+j)-1+j=0$ ges.: Lösung der Gl.

$$x^3 - 1 = 0 \qquad |: 1+j \neq 0$$

$x_1 = 1$ sieht man

$$x^3 - 1 = 0 = x^3 + 0x^2 + 0x - 1 = 0$$

	1	0	0	-1
	–	1	1	1
$x=1$	1	1	1	0

$\Rightarrow \quad x^2 + x + 1 = 0$

$$x_{2/3} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - 1} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{4}{4}}$$

$$= -\frac{1}{2} \pm \sqrt{-\frac{3}{4}} = -\frac{1}{2} \pm j\,\frac{\sqrt{3}}{2}$$

$x_1 = 1$, $x_2 = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$, $x_3 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$

Klausur <u>24.09.02</u> (A1)

ges.: $z^5 + 6z^4 + 13z^3 + 8z^2 + 48z + 104 = 0$

$z_1 = -3+2j$ (Nullstelle)

	1	6	13	8	48	+104
	–	$-3+2j$	-13	0	$-24+16j$	-104
$z_1 = -3+2j$	1	$3+2j$	0	8	$24+6j$	0
	—	$-3-2j$	0	0	$-24-16j$	
konj. kompl. $z_2 = -3-2j$	1	0	0	8	0	
	—	-2	4	-8		
$z_3 = -2$	1	-2	4	0		

$\Rightarrow z^3 + 8 = 0 \Rightarrow z_3 = -2$

$\Rightarrow z^2 - 2z + 4 = 0$

$$z_{4/5} = 1 \pm \sqrt{1-4} = 1 \pm \sqrt{-3} = 1 \pm \sqrt{3}\,j$$

$z_1 = -3+2j$

$z_2 = -3-2j$

$z_3 = -2$

$z_4 = 1 + \sqrt{3}\,j$

$z_5 = 1 - \sqrt{3}\,j$

23.09.07 (A2) Cramersche Regel

$$z_1 + (2-j)\cdot z_2 = 1+j$$
$$jz_1 + 3z_2 = 2-j$$

$$D = \begin{vmatrix} 1 & (2-j) \\ j & 3 \end{vmatrix} = 3 - (j(2-j)) = 3-(2j-j^2) = 3-(2j+1) = \underline{\underline{2-2j}}$$

$$D_{z_1} = \begin{vmatrix} 1+j & 2-j \\ 2-j & 3 \end{vmatrix} = 3(1+j) - ((2-j)^2) = 3+3j-(4-4j-1) = 3+3j-4+4j+1 = \underline{\underline{7j}}$$

$$D_{z_2} = \begin{vmatrix} 1 & 1+j \\ j & 2-j \end{vmatrix} = 2-j - (j\cdot(1+j)) = 2-j-(j+j^2) = 2-j-(j-1) = 2-j-j+1 = \underline{\underline{3-2j}}$$

$$z_1 = \frac{D_{z1}}{D} = \frac{7j}{2-2j} = \frac{7j\cdot(2+2j)}{(2-2j)(2+2j)} = \frac{14j+14j^2}{4+4} = \frac{14j-14}{8} = \underline{\underline{-\frac{7}{4}+\frac{7}{4}j}}$$

$$z_2 = \frac{D_{z2}}{D} = \frac{3-2j}{2-2j} = \frac{(3-2j)\cdot(2+2j)}{(2-2j)\cdot(2+2j)} = \frac{6-4j+6j-4j^2}{4+4} = \frac{10+2j}{8} = \underline{\underline{\frac{5}{4}+\frac{j}{4}}}$$

Quadratische wurzel von z_2:

$$\varphi = \arctan\left(\frac{Im}{Re}\right) = \arctan\left(\frac{\frac{1}{4}}{\frac{5}{4}}\right) = \arctan\left(\frac{4}{4\cdot 5}\right) = \arctan\left(\frac{1}{5}\right)$$

$$r = \sqrt{Re^2 + Im^2} = \sqrt{\left(\frac{5}{4}\right)^2 + \left(\frac{1}{4}\right)^2} = \sqrt{\frac{25}{16}+\frac{1}{16}} = \frac{1}{4}\sqrt{26}$$

$$z_{2,1} = \sqrt[2]{r}\cdot\left(\cos\frac{\arctan\left(\frac{1}{5}\right)}{2} + j\sin\frac{\arctan\frac{1}{5}}{2}\right) = 1{,}1236 + j\,0{,}1113$$

weil Quadratwurzel und Symetrie $\Rightarrow$ $z_{2,2} = -1{,}1236 - j\,0{,}1113$

Klausur
18.09.01 A1

geg.: $z_1 + z_2 = 4$

$z_1 \cdot z_2 = 8$

$z_1 = 4 - z_2$

$(4 - z_2) \cdot z_2 = 8$

$-z_2^2 + 4z_2 - 8 = 0$

$z_2^2 - 4z_2 + 8 = 0$

$z_{2_{1/2}} = 2 \pm \sqrt{4-8} = 2 \pm \sqrt{-4} = 2 \pm j\sqrt{4} = 2 \pm j2$

$r = \sqrt{2^2 + 2^2} = \sqrt{8}$

$\varphi_1 = \arctan\left(\frac{2}{2}\right) = \arctan 1 = \frac{\pi}{4}$

$\varphi_2 = \arctan\left(\frac{2}{-2}\right) = \arctan -1 = -\frac{\pi}{4}$

$z_1 = 2 + j2 = \sqrt{8} \cdot \left(\cos\left(\frac{\pi}{4}\right) + j\sin\left(\frac{\pi}{4}\right)\right)$

$z_2 = 2 - j2 = \sqrt{8} \cdot \left(\cos\left(-\frac{\pi}{4}\right) + j\sin\left(-\frac{\pi}{4}\right)\right)$

Klausur
13.03.01 A1

$z^4 + z^2 + 1 = 0$ alle Lösungen gesucht

Reduktion durch Substitution

$z^2 = t$

$\Rightarrow t^2 + t + 1 = 0$

$t_{1/2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{4}{4}} = -\frac{1}{2} \pm \sqrt{-\frac{3}{4}} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

$t_1 = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$ $\quad t_2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$

$z = \pm\sqrt{t}$

$z_1 = -\sqrt{-\frac{1}{2} + j\frac{\sqrt{3}}{2}}$ $\quad z_3 = -\sqrt{-\frac{1}{2} - j\frac{\sqrt{3}}{2}}$

$z_2 = +\sqrt{-\frac{1}{2} + j\frac{\sqrt{3}}{2}}$ $\quad z_4 = +\sqrt{-\frac{1}{2} - j\frac{\sqrt{3}}{2}}$

$r = \quad , \quad \varphi = \arctan \frac{Im}{re} = -\frac{\pi}{3}$

Fortsetzung 13.03.01

$z_1 = \cos\left(\frac{-\frac{\pi}{3}}{2}\right) + j\sin\left(\frac{-\frac{\pi}{3}}{2}\right) = \cos\left(\frac{\pi}{6}\right) + j\sin\left(-\frac{\pi}{6}\right) =$

$= \underline{\underline{\frac{\sqrt{3}}{2} - j\frac{1}{2}}}$

$z_2 = \cos\left(\frac{-\frac{\pi}{3} + 2\pi}{2}\right) + j\sin\left(\frac{-\frac{\pi}{3} + 2\pi}{2}\right) = \underline{\underline{-\frac{\sqrt{3}}{2} + j\frac{1}{2}}}$

$z_3 = \cos\left(\frac{\frac{\pi}{3}}{2}\right) + j\sin\left(\frac{\frac{\pi}{3}}{3}\right) = \underline{\underline{\frac{\sqrt{3}}{2} + j\frac{1}{2}}}$

$z_4 = \cos\left(\frac{\frac{\pi}{3} + 2\pi}{2}\right) + j\sin\left(\frac{\frac{\pi}{3} + 2\pi}{2}\right) = \underline{\underline{-\frac{\sqrt{3}}{2} - j\frac{1}{2}}}$

Klausur

20.09.05 A1 b)

$(-j)^{\frac{1}{3}} = ?$ mit Satz 12 S. 106

$z^{\frac{p}{q}} = r^{\frac{p}{q}}\left(\cos\left(\frac{p\varphi + kp2\pi}{q}\right) + j\sin\left(\frac{p\varphi + kp2\pi}{q}\right)\right)$

$r = \sqrt{im^2 + re^2} = \sqrt{1} = 1$

$\varphi = \arctan\left(\frac{Im}{re}\right) = \arctan\left(\frac{-1}{0}\right) \; ↯ \Rightarrow$ Quatrantenbetrachtung ergibt $\varphi = \frac{\pi}{2}$

$p = 1$

$q = 3$

$k = 0, 1, 2$

$k = 0$ (Hauptwert)

$z_H = \cos\left(\frac{\frac{\pi}{2}}{3}\right) + j\sin\left(\frac{\frac{\pi}{2}}{3}\right) = \cos\left(\frac{\pi}{6}\right) + j\sin\left(\frac{\pi}{6}\right)$

$z_H = \underline{\frac{\sqrt{3}}{2} + j\frac{1}{2}}$

$k = 1$

$z_1 = \cos\left(\frac{\frac{\pi}{2} + 2\pi}{3}\right) + j\sin\left(\frac{\frac{\pi}{2} + 2\pi}{3}\right) = \underline{\underline{-\frac{\sqrt{3}}{2} + j\frac{1}{2}}}$

$k = 2$

$z_2 = \cos\left(\frac{\frac{\pi}{2} + 4\pi}{3}\right) + j\sin\left(\frac{\frac{\pi}{2} + 4\pi}{3}\right) = \underline{\underline{0 - j}}$

Klausur
09.03.06 A1

$z^4 - 4z^2 + 36 = 0$

$t = z^2$

$\Rightarrow \quad t^2 - 4t + 36 = 0$

$t_{1/2} = 2 \pm \sqrt{4-36} = 2 \pm \sqrt{-32} = 2 \pm j\sqrt{32}$

$t_1 = 2 + j\sqrt{32}$
$t_2 = 2 - j\sqrt{32}$

$z_{1/2} = \sqrt{t_1} \quad , \quad z_{2/3} = \sqrt{t_2}$

$r_1 = r_2 = \sqrt{re^2 + im^2} = \sqrt{2^2 + \sqrt{32}^2} = \sqrt{36} = 6$

$\varphi_1 = \arctan \frac{\sqrt{32}}{2} = 1{,}2309$

$\varphi_2 = \arctan \frac{-\sqrt{32}}{2} = -1{,}2309$

$z_1 = \sqrt{6} \cdot \left[\cos\left(\frac{1{,}2309}{2}\right) + j\sin\left(\frac{1{,}2309}{2}\right)\right] = 2 + j1{,}4142$

$z_2 = \sqrt{6} \cdot \left[\cos\left(\frac{1{,}2309 + 2\pi}{2}\right) + j\sin\left(\frac{1{,}2309 + 2\pi}{2}\right)\right] = -2 - j1{,}4142$

$z_3 = \sqrt{6} \cdot \left[\cos\left(\frac{-1{,}2309}{2}\right) + j\sin\left(\frac{-1{,}2309}{2}\right)\right] = 2 - j1{,}4142$

$z_4 = \sqrt{6} \cdot \left[\cos\left(\frac{-1{,}2309 + 2\pi}{2}\right) + j\sin\left(\frac{-1{,}2309 + 2\pi}{2}\right)\right] = -2 + j1{,}4142$