09. 03. 06

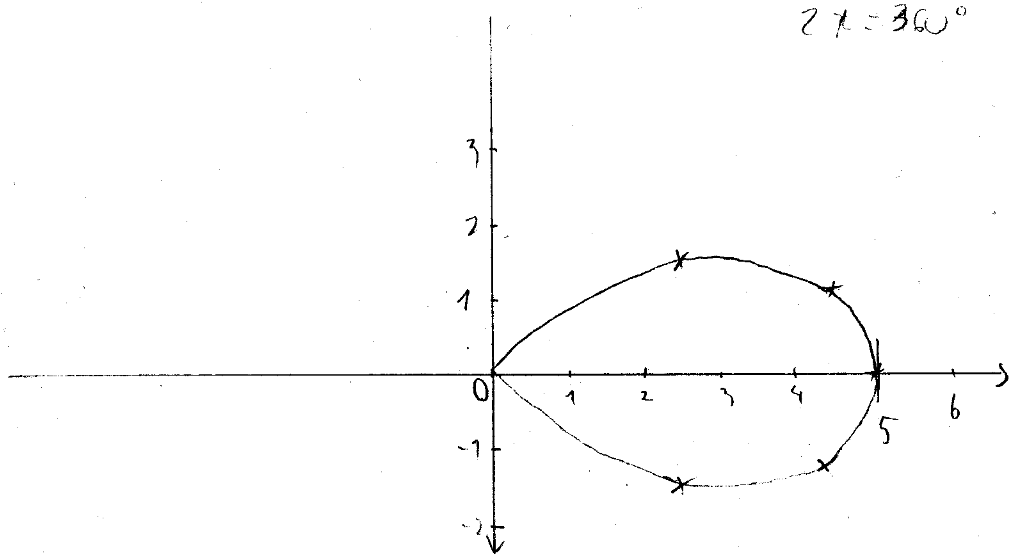
Aufgabe 3:

$$r(\varphi) = 10\cos(\varphi) - \frac{5}{\cos(\varphi)}$$

a) wertetabelle + zeichnung (schrittweite $\frac{\pi}{12}$) $\hat{=}\ 15°$

φ	$-\frac{\pi}{4}$ $\left(= -\frac{3\pi}{12}\right)$	$-\frac{2\pi}{12}$	$-\frac{\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{2\pi}{12}$	$\frac{3\pi}{12}$					
r	0	2,89	4,48	5	4,48	2,89	0					
	-45	-30	-15	0	15	30	45					

$2\pi = 360°$



A = ges

$$A = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} r^2\,d\varphi = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(10\cos(\varphi) - \frac{5}{\cos(\varphi)}\right)^2 d\varphi$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 100\cos^2(\varphi) - 2\cdot 10\cos(\varphi) + \frac{25}{\cos(\varphi)}\,d\varphi$$

$$= \underbrace{100\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2(\varphi)\,d\varphi}_{I_{II},\ \text{Typ 4}} - \underbrace{20\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos(\varphi)\,d\varphi}_{I_{III}\ \text{Stammintegral}} + \underbrace{25\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{\cos(\varphi)}\,d\varphi}_{I_1}$$

2)

$$\boxed{I_1 = \int \frac{1}{\cos(x)}\,dx} = \int \frac{\sin x}{\sin x \cdot \cos x}\,dx \qquad u = \sin x$$

$$\frac{du}{dx} = \cos x \;\rightsquigarrow\; dx = \frac{du}{\cos x}$$

$$= \int \frac{u}{u \cdot \cos^2 x}\,du \qquad \Big| \quad \cos^2 x = 1 - \sin^2 x$$

$$\sin x = u \quad \Rightarrow \cos^2 x = 1 - u^2$$

$$= \int \frac{u}{u\cdot(1-u^2)}\,du = \int \frac{1}{1-u^2}\,du$$

$$= \operatorname{arctanh}(u) = \operatorname{arctanh}(\sin x) = \frac{1}{2}\left[\ln\left|\frac{1+\sin x}{1-\sin x}\right|\right]$$

$$= \frac{1}{2}\Big[\ln|1+\sin x| - \ln|1-\sin x|\Big] = \tfrac{1}{2}\ln|1+\sin x| - \tfrac{1}{2}\ln|1-\sin x|$$

$$A = \frac{1}{2}\Big[100\left(x - \sin x\cdot\cos x\right) + 20\sin x + 25\operatorname{arctanh}(\sin x)\Big]\Bigg|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$A = \frac{1}{2}\Bigg[\Big\{100\Big(\frac{\pi}{4} - \underbrace{\sin\Big(\frac{\pi}{4}\Big)}_{\frac{1}{2}\sqrt{2}}\cdot\underbrace{\cos\Big(\frac{\pi}{4}\Big)}_{\frac{1}{2}\sqrt{2}}\Big) + 20\cdot\underbrace{\sin\Big(\frac{\pi}{4}\Big)}_{\frac{\sqrt{2}}{2}} + 25\operatorname{arctanh}\Big(\underbrace{\sin\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}\Big)\Big\}$$

$$- 100\Big(-\frac{\pi}{4} - \underbrace{\sin\Big(-\frac{\pi}{4}\Big)}_{-\frac{\sqrt{2}}{2}}\cdot\underbrace{\cos\Big(-\frac{\pi}{4}\Big)}_{\frac{\sqrt{2}}{2}}\Big) + 20\underbrace{\sin\Big(-\frac{\pi}{4}\Big)}_{-\frac{\sqrt{2}}{2}} + 25\operatorname{arctanh}\Big(\underbrace{\sin\Big(-\frac{\pi}{4}\Big)}_{-\frac{\sqrt{2}}{2}}\Big)\Big\}\Bigg]$$

$A =$

$$A = 50\cdot\Bigg[\Big(\frac{\pi}{4} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} + 20\frac{\sqrt{2}}{2} + 25\operatorname{arctanh}\Big(\frac{\sqrt{2}}{2}\Big)\Big) - \Big(\frac{\pi}{4} - \frac{-\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} - 20\frac{\sqrt{2}}{2} + 25\operatorname{arctanh}\Big(-\frac{\sqrt{2}}{2}\Big)\Big)\Bigg]$$

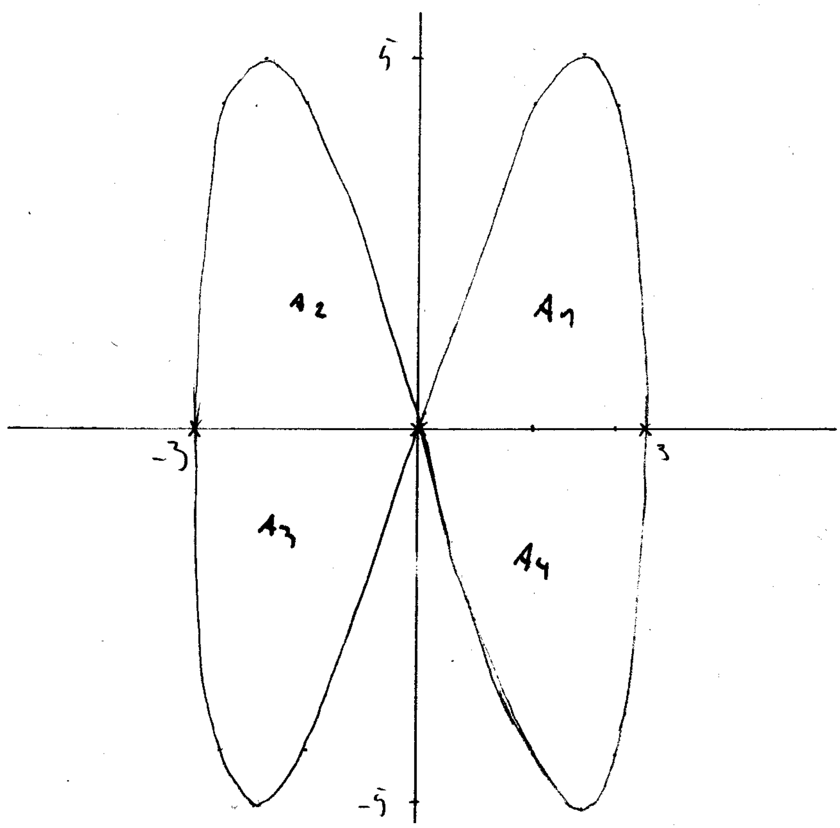
$$A = 50\cdot\Bigg[\Big(\frac{\pi}{4} - \frac{1}{2} + 10\sqrt{2} + 25\operatorname{arctanh}\Big(\frac{\sqrt{2}}{2}\Big)\Big) - \Big(\frac{\pi}{4} + \frac{1}{2} - 10\sqrt{2} + 25\operatorname{arctanh}\Big(-\frac{\sqrt{2}}{2}\Big)\Big)\Bigg]$$

A3 09.02.05

$x(t) = 3\sin(t)$
$y(t) = 5 \cdot \sin(2t)$

a) wertetabelle

t	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	$\frac{6\pi}{6}$	$\frac{7\pi}{6}$	$\frac{8\pi}{6}$	$\frac{9\pi}{6}$	$\frac{10\pi}{6}$	$\frac{11\pi}{6}$	$\frac{12\pi}{6}$
x	0	1,5	2,6	3	2,6	1,5	0	-1,5	-2,6	-3	-2,6	-1,5	0
y	0	4,3	4,3	0	-4,3	-4,3	0	4,3	4,3	0	-4,3	-4,3	0



$A = A_1 \cdot 4 \quad (A_1 = A_2 = A_3 = A_4)$

$$A_1 = \int_0^{\frac{\pi}{2}} y(t) \cdot \dot{x}(t)\, dt \qquad \dot{x}(t) = 3\cos(t)$$

$$A_1 = \int_0^{\frac{\pi}{2}} 5 \cdot \sin(2t) \cdot 3\cos(t)\, dt = 15 \int_0^{\frac{\pi}{2}} \sin(2t) \cdot \cos t$$

$$A_1 = 15 \int_0^{\frac{\pi}{2}} 2\sin(t) \cdot \cos(t) \cdot \cos(t)\, dt = 30 \cdot \int_0^{\frac{\pi}{2}} \sin(t) \cdot \cos^2(t)\, dt$$

4)

$u = \cos t$

$\frac{du}{dt} = -\sin(t) \rightsquigarrow dt = \frac{du}{-\sin(t)}$

$$A_1 = 30 \int_0^{\frac{\pi}{2}} \sin t \cdot u^2 \cdot \frac{du}{-\sin t} = -30 \int_0^{\frac{\pi}{2}} u^2 du = -30\left(\frac{1}{3} u^3\right)\Big|_0^{\frac{\pi}{2}}$$

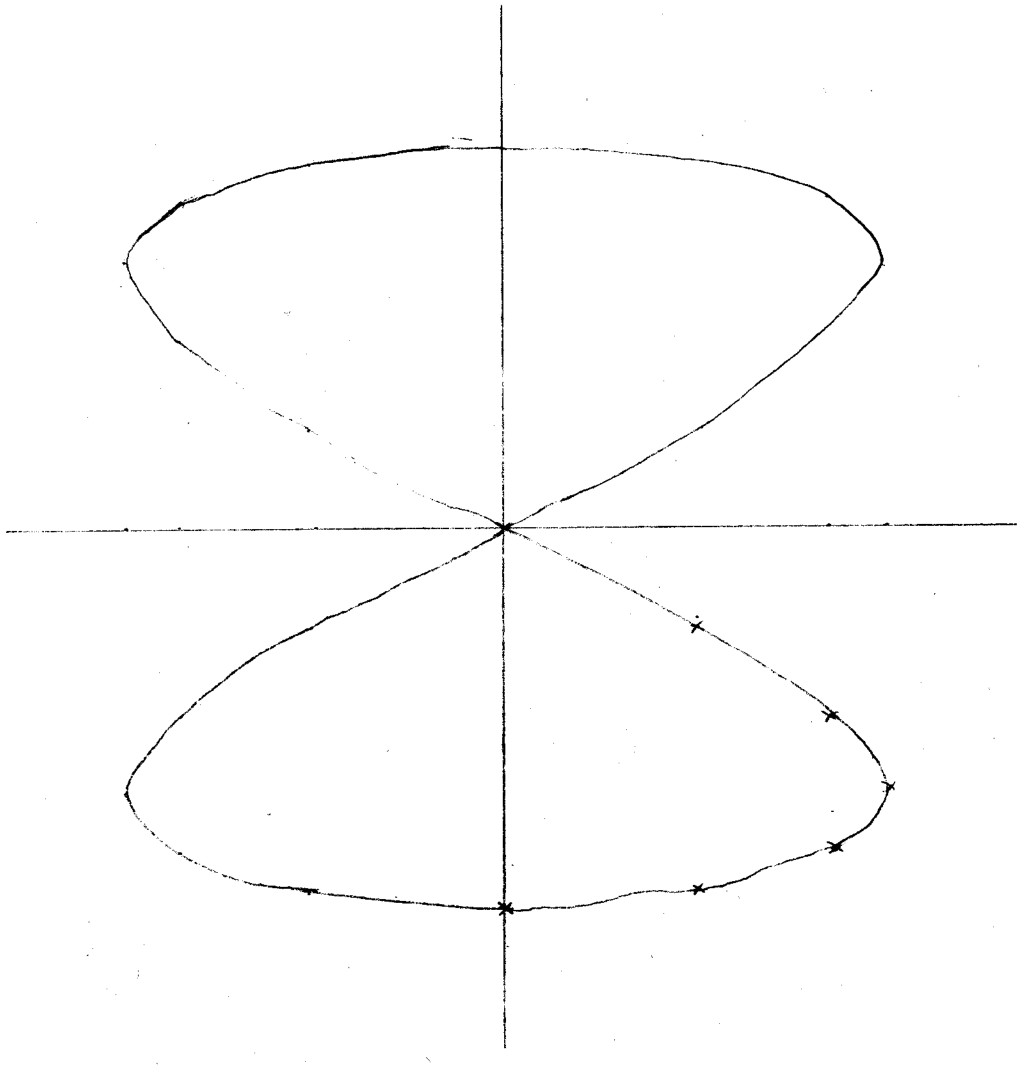
$$A_1 = \left[-10 \cdot \cos^3(t)\right]_0^{\frac{\pi}{2}} = 0 - (-10 \cdot \cos^3(0)) = 10$$

$A = A_1 \cdot 4 = 40$ FE

28.09.04 A4

a) ges.: $x(t) = \sin(2t)$
$y(t) = \cos(t)$

t	π	$\frac{13\pi}{12}$	$\frac{14\pi}{12}$	$\frac{15\pi}{12}$	$\frac{16\pi}{12}$	$\frac{17\pi}{12}$	$\frac{18\pi}{12}$	$\frac{19\pi}{12}$	$\frac{20\pi}{12}$	$\frac{21\pi}{12}$	$\frac{22\pi}{12}$	$\frac{23\pi}{12}$	$\frac{24\pi}{12}$
x	0	0,5	0,87	1	0,87	0,5	0	-0,5	-0,87	-1	-0,87	-0,5	0
y	-1	-0,97	-0,87	-0,71	-0,5	-0,26	0	0,26	0,5	0,71	0,87	0,97	1



b) geg.: $x(t) = \sin(t)$ $\quad \dot{x}(t) = \cos(t)$

$y(t) = \cos(2t)$

ges.: $A_G = ?$

$A_G = 4 \cdot A$

$$A = \int_0^{\frac{\pi}{2}} y(t) \cdot \dot{x}(t)\, dt = \int_0^{\frac{\pi}{2}} \cos(2t) \cdot \cos(t)\, dt \qquad \Big|\cos(2t) = 2\cos^2 t - 1$$

$$A = \int_0^{\frac{\pi}{2}} (2\cos^2 t - 1)\cos(t)\, dt = \int_0^{\frac{\pi}{2}} 2\cos^3 t - \cos t\, dt$$

$$A = 2\int_0^{\frac{\pi}{2}} \cos^3 t\, dt - \int_0^{\frac{\pi}{2}} \cos t\, dt = 2\int_0^{\frac{\pi}{2}} \frac{1}{4}(3\cos(t) + \cos(3t)) - \sin t$$

$$A = 2\int_0^{\frac{\pi}{2}} \frac{3}{4}\cos(t) + \frac{1}{4}\cos(3t)\, dt - \sin t = \frac{6}{4}\int_0^{\frac{\pi}{2}} \cos(t)\, dt + \frac{2}{4}\int_0^{\frac{\pi}{2}} \cos(3t)\, dt - \sin t$$

$u = 3t$

$\frac{du}{dt} = 3 \rightsquigarrow dt = \frac{du}{3}$

$$A = \frac{6}{4}\int_0^{\frac{\pi}{2}} \cos(t)\, dt + \frac{1}{12}\int_0^{\frac{\pi}{2}} \cos(u)\, du - \sin t$$

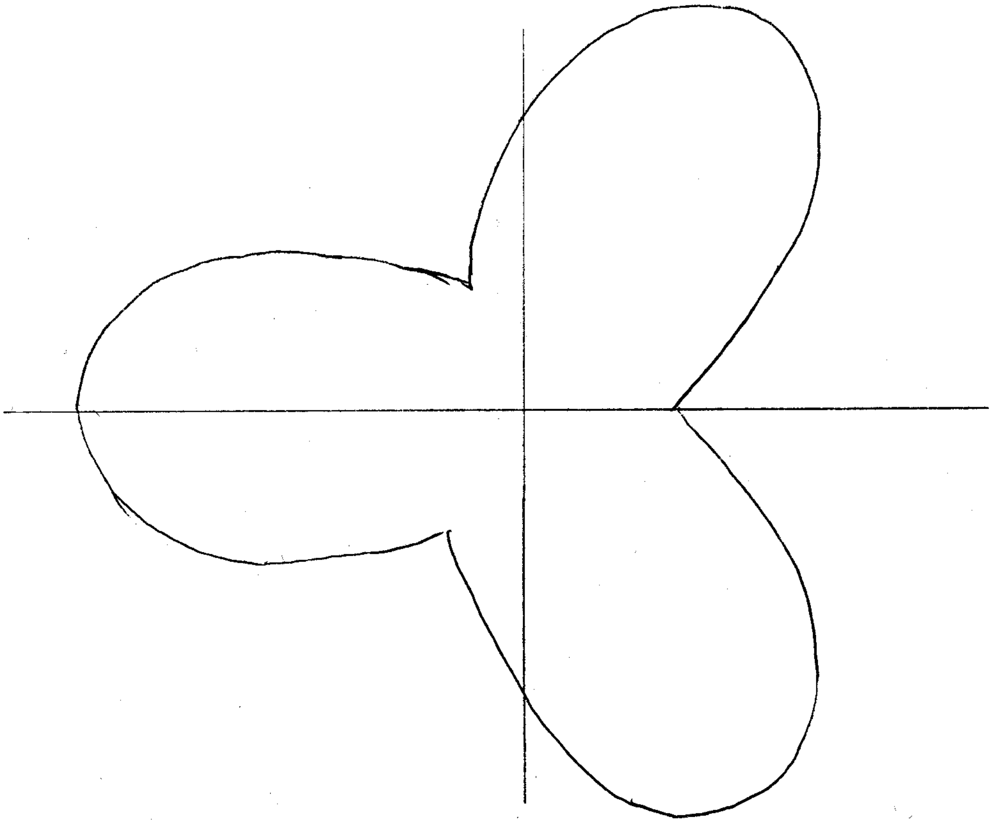
$$A = \frac{6}{4}\sin(t)\Big|_0^{\frac{\pi}{2}} + \frac{1}{6}\sin(3t)\Big|_0^{\frac{\pi}{2}} - \sin(t)\Big|_0^{\frac{\pi}{2}}$$

$$A = \left[\frac{6}{4}\sin(t) - \frac{4}{4}\sin(t) + \frac{1}{6}\sin(3t)\right]_0^{\frac{\pi}{2}} = \left[\frac{1}{2}\sin(t) + \frac{1}{6}\sin(3t)\right]_0^{\frac{\pi}{2}} = \frac{1}{3}$$

$A_G = 4 \cdot A = \frac{4}{3}$

6/ 11. 02. 04

(43) $r(\varphi) = 2 - \cos(3\varphi)$



$$A = \frac{1}{2}\int_0^{2\pi} r(\varphi)^2 \, d\varphi = \frac{1}{2}\left(\int_0^{2\pi} 4 - 4\cos(3\varphi) + \cos^2(3\varphi) \, d\varphi\right)$$

$$A = \frac{1}{2}\left(\underbrace{\int_0^{2\pi} 4 \, d\varphi}_{I_1} \underbrace{- \int_0^{2\pi} 4\cos(3\varphi)\, d\varphi}_{I_2} + \underbrace{\int_0^{2\pi} \cos^2(3\varphi)\, d\varphi}_{I_3}\right)$$

$$I_1 = \int 4 \, d\varphi = \underline{\underline{4\varphi}}$$

$$I_2 = -\int 4\cos(3\varphi)\, d\varphi = -\int 4\cos(t)\frac{dt}{3} = -\frac{4}{3}\sin t = \underline{\underline{-\frac{4}{3}\sin(3\varphi)}}$$

$$t \rightarrow \frac{dt}{d\varphi} = 3 \leadsto d\varphi = \frac{dt}{3}$$

$$I_3 = \int \cos^2(3\varphi)\, d\varphi = \int \cos^2(t)\frac{dt}{3} = \frac{1}{3}\left[\frac{1}{2}t - \frac{1}{4}\sin(2t)\right] = \frac{1}{6}3\varphi - \frac{1}{12}\sin(6\varphi)$$

$$I_3 = \frac{1}{2}\varphi - \frac{1}{12}\sin(6\varphi)$$

$$A = \left[\frac{1}{2}\left(I_1 + I_2 + I_3\right)\right]_0^{2\pi} = \left[\frac{1}{2}\left(4\varphi - \frac{4}{3}\sin(3\varphi) + \frac{1}{2}\varphi - \frac{1}{12}\sin(6\varphi)\right)\right]_0^{2\pi}$$

$$A = \left[2\varphi - \frac{4}{6}\sin(3\varphi) + \frac{1}{4}\varphi - \frac{1}{24}\sin(6\varphi)\right]_0^{2\pi}$$

$$A = \left[\frac{9}{4}\varphi - \frac{4}{6}\sin(3\varphi) - \frac{1}{24}\sin(6\varphi)\right]_0^{2\pi} = \frac{9}{4}\varphi\Big|_0^{2\pi} = \frac{18}{4}\pi = \frac{9}{2}\pi$$

$$\frac{9\pi}{2}$$

12.02.03

A3

$k_1: \quad r(\varphi) = 1 + \sin^2(2\varphi)$

$k_2: \quad r(\varphi) = 1$

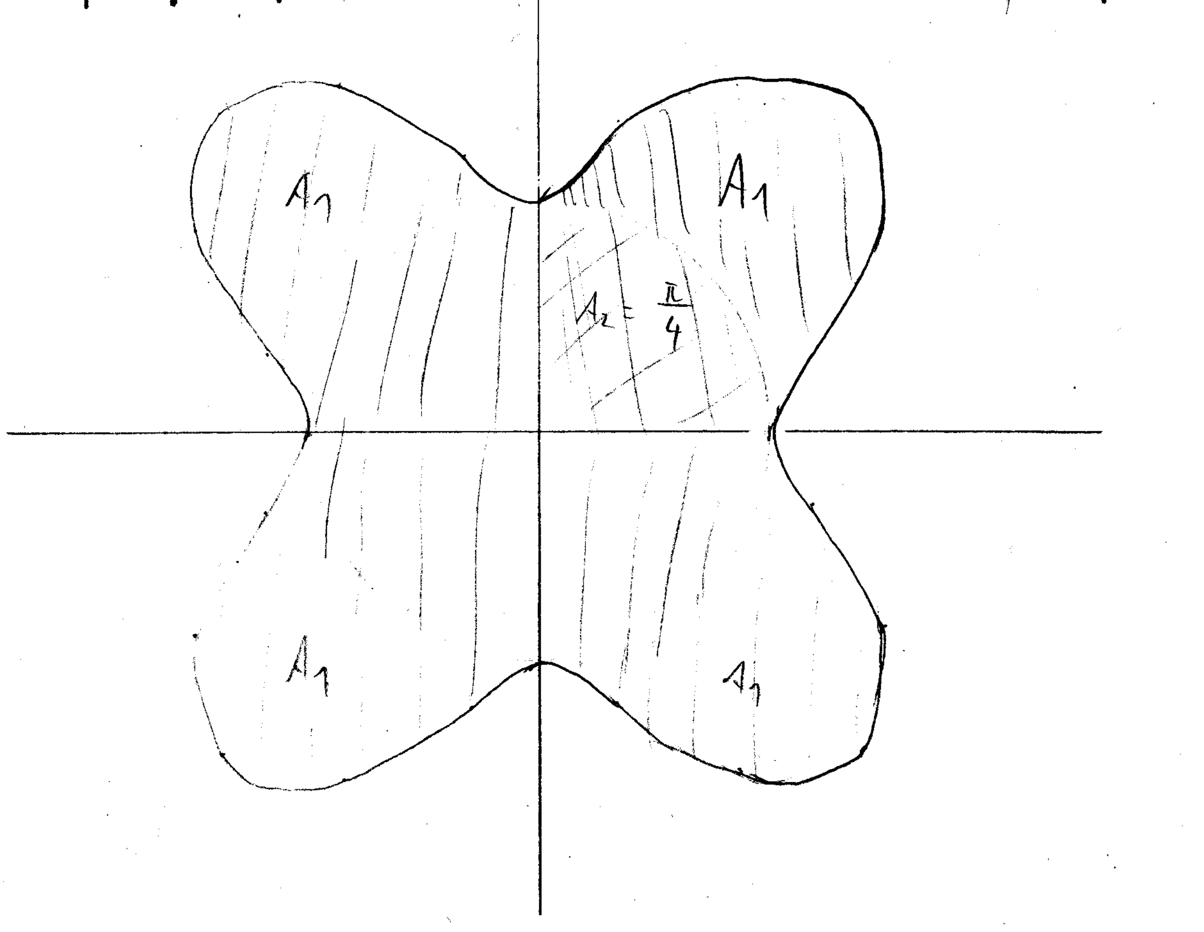
a) k_2 ist ein Kreis mit $r = 1$

k_1:

φ	0	$\frac{\pi}{12}$	$\frac{2\pi}{12}$	$\frac{3\pi}{12}$	$\frac{4\pi}{12}$	$\frac{5\pi}{12}$	$\frac{6\pi}{12}$	$\frac{7\pi}{12}$	$\frac{8\pi}{12}$	$\frac{9\pi}{12}$	$\frac{10\pi}{12}$	$\frac{11\pi}{12}$	$\frac{12}{12}\pi$
r	1	1,25	1,75	2	1,75	1,25	1	1,25	1,75	2	1,75	1,25	1

k_1:

φ	$\frac{13\pi}{12}$	$\frac{14\pi}{12}$	$\frac{15\pi}{12}$	$\frac{16\pi}{12}$	$\frac{17\pi}{12}$	$\frac{18\pi}{12}$	$\frac{19\pi}{12}$	$\frac{20\pi}{12}$	$\frac{21\pi}{12}$	$\frac{22\pi}{12}$	$\frac{23\pi}{12}$	$\frac{24\pi}{12}$
r	1,25	1,75	2	1,75	1,25	1	1,25	1,75	2	1,75	1,25	1



$$A = 4\cdot\frac{1}{2}\int_0^{\frac{\pi}{2}} \left(1+\sin^2(2\varphi)\right)^2 d\varphi - \pi = 2\int_0^{\frac{\pi}{2}} 1 + 2\sin(2\varphi) + \sin^4(2\varphi)\, d\varphi - \pi$$

$$A = 2\int_0^{\frac{\pi}{2}} 1 + 2\cdot\frac{1}{2}\left(1-\cos(4\varphi)\right) + \frac{1}{8}\left(\cos(8\varphi) - 4\cos(4\varphi) + 3\right) d\varphi - \pi$$

$$A = 2\int_0^{\frac{\pi}{2}} \left[1 + 1 - \cos(4\varphi) + \frac{1}{8}\cos(8\varphi) - \frac{1}{2}\cos(4\varphi) + \frac{3}{8}\right] d\varphi - \pi$$

$$A = 2\int_0^{\frac{\pi}{2}} \left[\frac{19}{8} - \cos(4\varphi) + \frac{1}{8}\cos(8\varphi) - \frac{1}{2}\cos(4\varphi)\right] - \pi$$

$$A = 2\left[\frac{19}{8}\varphi - \frac{1}{4}\sin(4\varphi) + \frac{1}{64}\sin(8\varphi) - \frac{1}{8}\sin(4\varphi)\right]\Big|_0^{\frac{\pi}{2}} - \pi$$

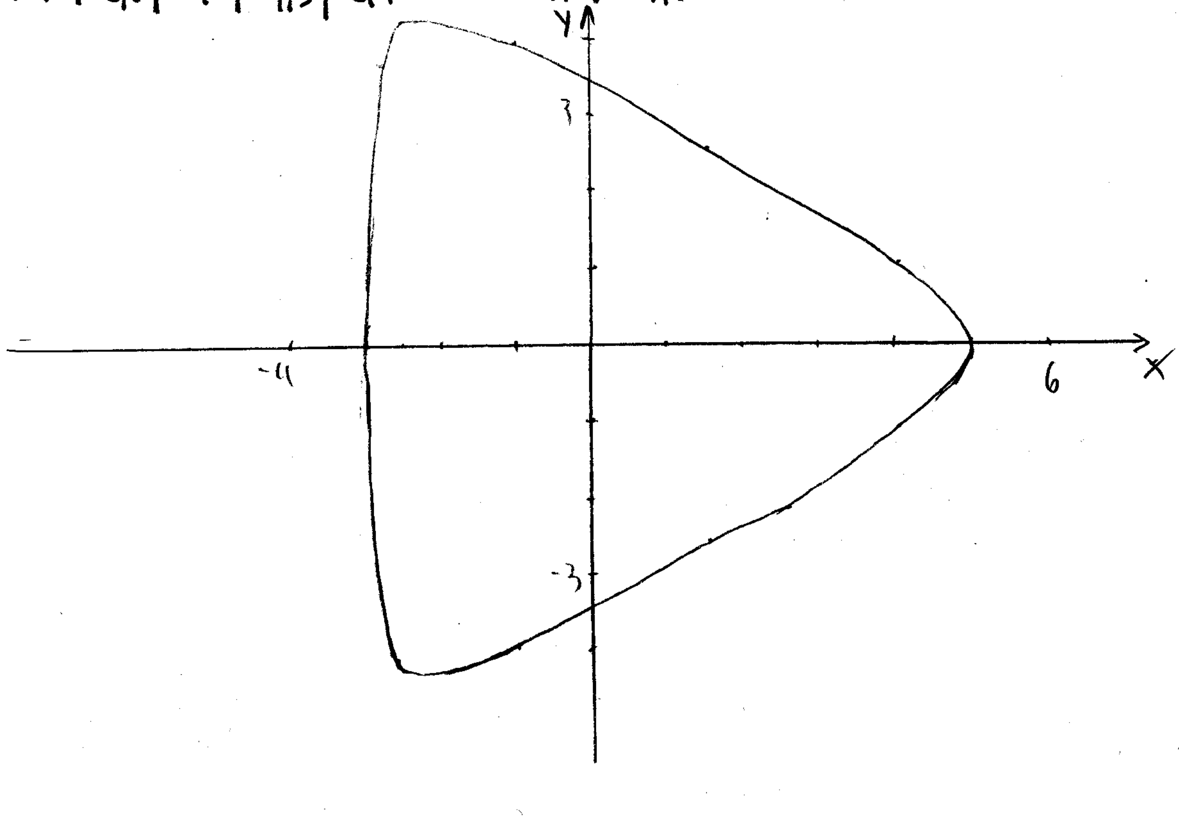
$$A = 2\left[\left(\frac{19}{8}\cdot\frac{\pi}{2} - 0 + 0 - 0\right) - (0)\right] - \pi$$

$$A = \frac{19}{8}\pi - \pi = \frac{19}{8}\pi - \frac{8}{8}\pi = \underline{\underline{\frac{11}{8}\pi}}$$

24.09.02
A3

$x(t) = 4\cos(t) + \cos(2t)$
$y(t) = 4\sin(t) - \sin(2t)$

t	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	$\frac{6\pi}{6}$	$\frac{7\pi}{6}$	$\frac{8\pi}{6}$	$\frac{9\pi}{6}$	$\frac{10\pi}{6}$	$\frac{11\pi}{6}$	$\frac{12\pi}{6}$
x	5	4	1,5	-1	-2,5	-3	-3	-3	-2,5	-1	1,5	4	5
y	0	1,1	2,6	4	4,3	2,9	0	-2,9	-4,3	-4	-2,6	-1,1	0



$$A = 2\cdot\frac{1}{2}\int_0^{\pi} (x\cdot\dot{y}) - (\dot{x}\cdot y)\,dt \qquad \begin{array}{ll} x = 4\cos(t) + \cos(2t) & y = 4\sin t - \sin(2t) \\ \dot{x} = -4\sin t - 2\sin(2t) & \dot{y} = 4\cos t - 2\cos(2t) \end{array}$$

$$A = \int_0^{\pi} \Big[\big(4\cos t + \cos(2t)\big)\cdot\big(4\cos t - 2\cos(2t)\big) - \Big(\big(-4\sin t - 2\sin(2t)\big)\cdot\big(4\sin t - \sin(2t)\big)\Big)\Big]\,dt$$

$$A = \int_0^{\pi} 16\cos^2 t - 8\cos t\cos(2t) + 4\cos t\cos 2t - 2\cos^2(2t) - \big(-16\sin^2 t + 4\sin t\cdot\sin(2t) - 8\sin(t)\sin(2t) + 2\sin^2(2t)\big)\,dt$$

$$A = \int_0^{\pi} 16\cancel{\cos^2 t} - 4\cos t\cos(2t) - 2\cancel{\cos^2(2t)} + 16\cancel{\sin^2 t} + 4\sin t\sin(2t) - 2\cancel{\sin^2(2t)}\,dt$$

$$A = \int_0^{\pi} 16 - 2 - 4\cos t\cos(2t) + 4\sin(t)\sin(2t)\,dt$$

$$A = \int_0^{\pi} 14 - 4\underbrace{\big(\cos t\cos 2t - \sin(t)\sin(2t)\big)}_{\text{Hilfestellung: } = \cos(3t)}\,dt$$

$$A = \int_0^{\pi} 14 - 4\cos(3t)\,dt = \Big[14t - \frac{4}{3}\sin(3t)\Big]_0^{\pi}$$

$$A = 14\pi - 0 - 0 = \underline{\underline{14\pi \text{ FE}}}$$

06.02.02
A3

$r(\varphi) = \cos(2\varphi) + 0{,}5$

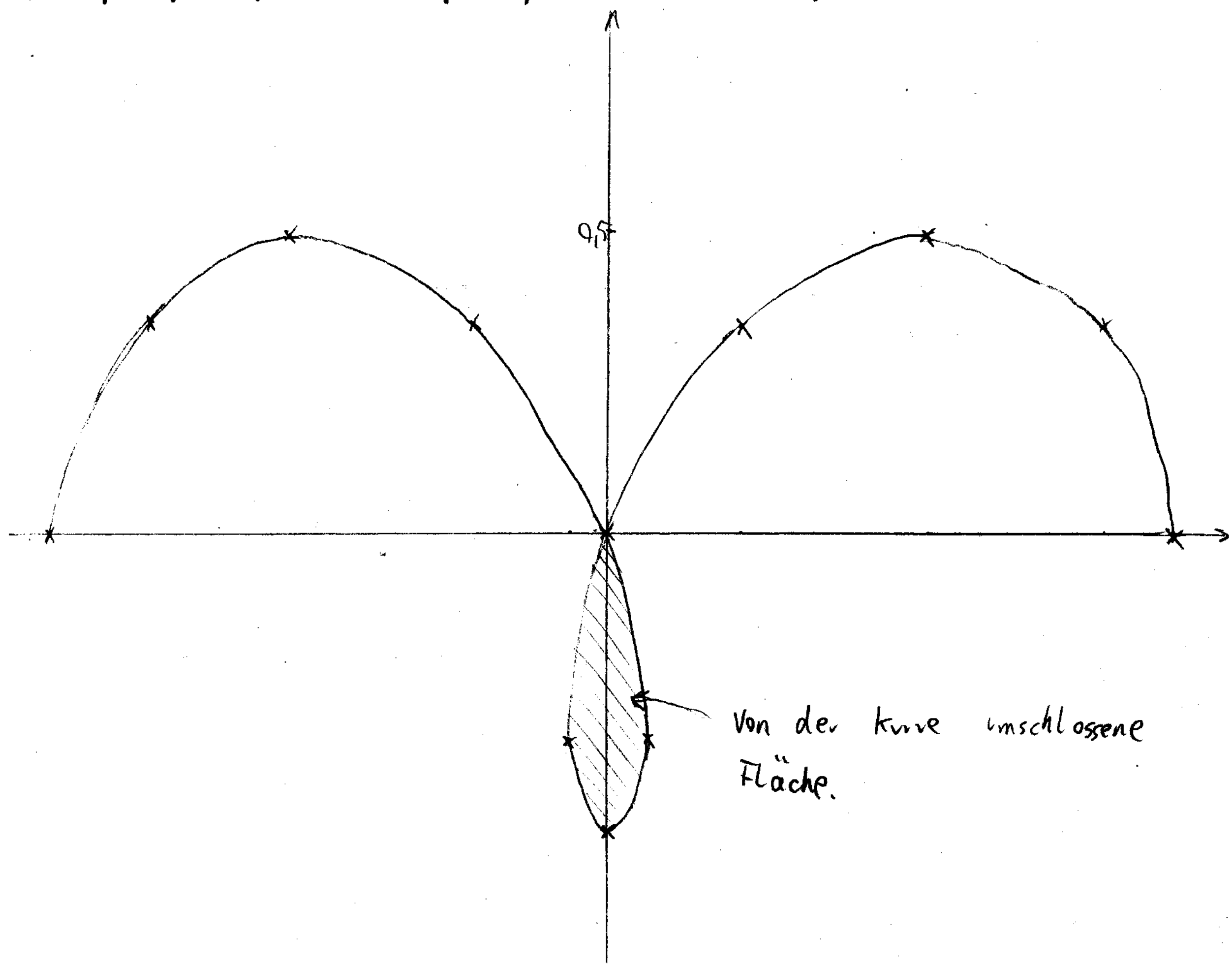
φ	0	$\frac{\pi}{12}$	$\frac{2\pi}{12}$	$\frac{3\pi}{12}$	$\frac{4\pi}{12}$	$\frac{5\pi}{12}$	$\frac{6\pi}{12}$	$\frac{7\pi}{12}$	$\frac{8\pi}{12}$	$\frac{9\pi}{12}$	$\frac{10\pi}{12}$	$\frac{11\pi}{12}$	$\frac{12\pi}{12}$
r	1,5	1,4	1	0,5	0	-0,4	-0,5	-0,4	0	0,5	1	1,4	1,5

Umrechnung der Wertetabelle in Zeichengerechten Maßstab

$x = r(\varphi) \cdot \cos(\varphi) \cdot 5 \frac{cm}{LE}$

$y = r(\varphi) \cdot \sin(\varphi) \cdot 8 \frac{cm}{LE}$

φ	0	$\frac{\pi}{12}$	$\frac{2\pi}{12}$	$\frac{3\pi}{12}$	$\frac{4\pi}{12}$	$\frac{5\pi}{12}$	$\frac{6\pi}{12}$	$\frac{7\pi}{12}$	$\frac{8\pi}{12}$	$\frac{9\pi}{12}$	$\frac{10\pi}{12}$	$\frac{11\pi}{12}$	$\frac{12\pi}{12}$
x	7,5	6,6	4,3	1,8	0	-0,5	0	0,5	0	-1,8	-4,3	-6,6	-7,5
y	0	2,8	4	2,8	0	-2,8	-4	-2,8	0	2,8	4	2,8	0



b) - Steigung gesucht in Abhängigkeit von φ
- Senkrechte Tangenten

$$r = \cos(2\varphi) + 0{,}5 = 2\cos^2\varphi - 1 + 0{,}5$$
$$\dot r = -2\sin(2\varphi) = -4\sin\varphi\cdot\cos\varphi$$

$$y' = \frac{\dot y}{\dot x} = \frac{\dot r\sin\varphi + r\cos\varphi}{\dot r\cos\varphi - r\sin\varphi}$$

$$x = \left(\cos(2\varphi)+0{,}5\right)\cdot\cos(\varphi) = \cos(2\varphi)\cos\varphi + 0{,}5\cos\varphi$$
$$y = \left(\cos(2\varphi)+0{,}5\right)\cdot\sin(\varphi) = \cos(2\varphi)\sin(\varphi) + 0{,}5\sin\varphi$$

$$y' = \frac{-4\sin\varphi\cos\varphi\sin\varphi + \left(2\cos^2\varphi-\frac{1}{2}\right)\cdot\cos\varphi}{-4\sin\varphi\cos\varphi\cos\varphi - \left(2\cos^2\varphi-\frac{1}{2}\right)\sin\varphi} = \frac{-4\sin^2\varphi\cos\varphi + 2\cos^3\varphi - \frac{1}{2}\cos\varphi}{-4\sin\varphi\cos^2\varphi - 2\cos^2\varphi\sin\varphi + \frac{1}{2}\sin\varphi}$$

$$y' = \frac{-4(1-\cos^2\varphi)\cos\varphi + 2\cos^3\varphi - \frac{1}{2}\cos\varphi}{-4(1-\sin^2\varphi)\sin\varphi - 2\cos^2\varphi\sin\varphi\frac{1}{2}\sin\varphi} = \frac{-4\cos\varphi + 4\cos^3\varphi + 2\cos^3\varphi - \frac{1}{2}\cos\varphi}{-4\sin\varphi + 4\sin^3\varphi - 2\cos^2\varphi\sin\varphi + \frac{1}{2}\sin\varphi}$$

$$y' = \frac{-4\cos\varphi + 6\cos^3\varphi - \frac{1}{2}\cos\varphi}{-4\sin\varphi + 4\sin^3\varphi - (2-2\sin^2\varphi)\sin\varphi + \frac{1}{2}\sin\varphi} = \frac{6\cos^3\varphi - \frac{9}{2}\cos\varphi}{6\sin^3\varphi - \frac{11}{2}\sin\varphi}$$

Senkrechte Tangenten für Zähler $\neq 0$ $\wedge$ Nenner $= 0$

Nenner: $\sin\varphi\left(6\sin^2\varphi - \frac{11}{2}\right)$

1. $\sin\varphi = 0 \Rightarrow \varphi_1 = 0 \vee \varphi_2 = \pi$
2. $6\sin^2\varphi - \frac{11}{2} = 0$

$$\sin^2\varphi = \frac{11}{12} \curvearrowright \sin\varphi = \sqrt{\frac{11}{12}} \curvearrowright \varphi = \arcsin\sqrt{\frac{11}{12}}$$

$\varphi_3 = 1{,}278$

$\varphi_4 = 1{,}864$ $\Rightarrow$ wegen Symetrie von π φ_3 abziehen

12) Berechnung der Fläche

Grenzen anhand des Schnittpunktes mit der x-Achse gewählt $\varphi_{oben} = \frac{2\pi}{3}$

$\varphi_{unten} = \frac{\pi}{3}$

$$A = \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} r^2 \, d\varphi = \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \cos^2(2\varphi) + \cos(2\varphi) + \frac{1}{4} \, d\varphi$$

Substitution: $s = 2\varphi$

$$\frac{ds}{d\varphi} = 2 \curvearrowright d\varphi = \frac{ds}{2}$$

$$A = \underbrace{\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \cos^2(s)\frac{ds}{2}}_{\text{Typ 4}} + \cos(s)\frac{ds}{2}\, ds + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{1}{4}\, d\varphi$$

$$A = \frac{1}{2}\left[\frac{1}{2}\left[\frac{1}{2}s + \frac{1}{4}\sin(2s)\right] + \sin s + \frac{1}{4}\varphi\right]_{\frac{\pi}{3}}^{\frac{2\pi}{3}}$$

$$A = \left[\frac{1}{2}\frac{1}{4}2\varphi + \frac{1}{16}\sin(4\varphi) + \frac{1}{2}\sin(2\varphi) + \frac{1}{8}\varphi\right]_{\frac{\pi}{3}}^{\frac{2\pi}{3}}$$

$$A = \left[\frac{1}{16}\sin(4\varphi) + \frac{1}{4}\sin(2\varphi) + \frac{3}{8}\varphi\right]_{\frac{\pi}{3}}^{\frac{2\pi}{3}}$$

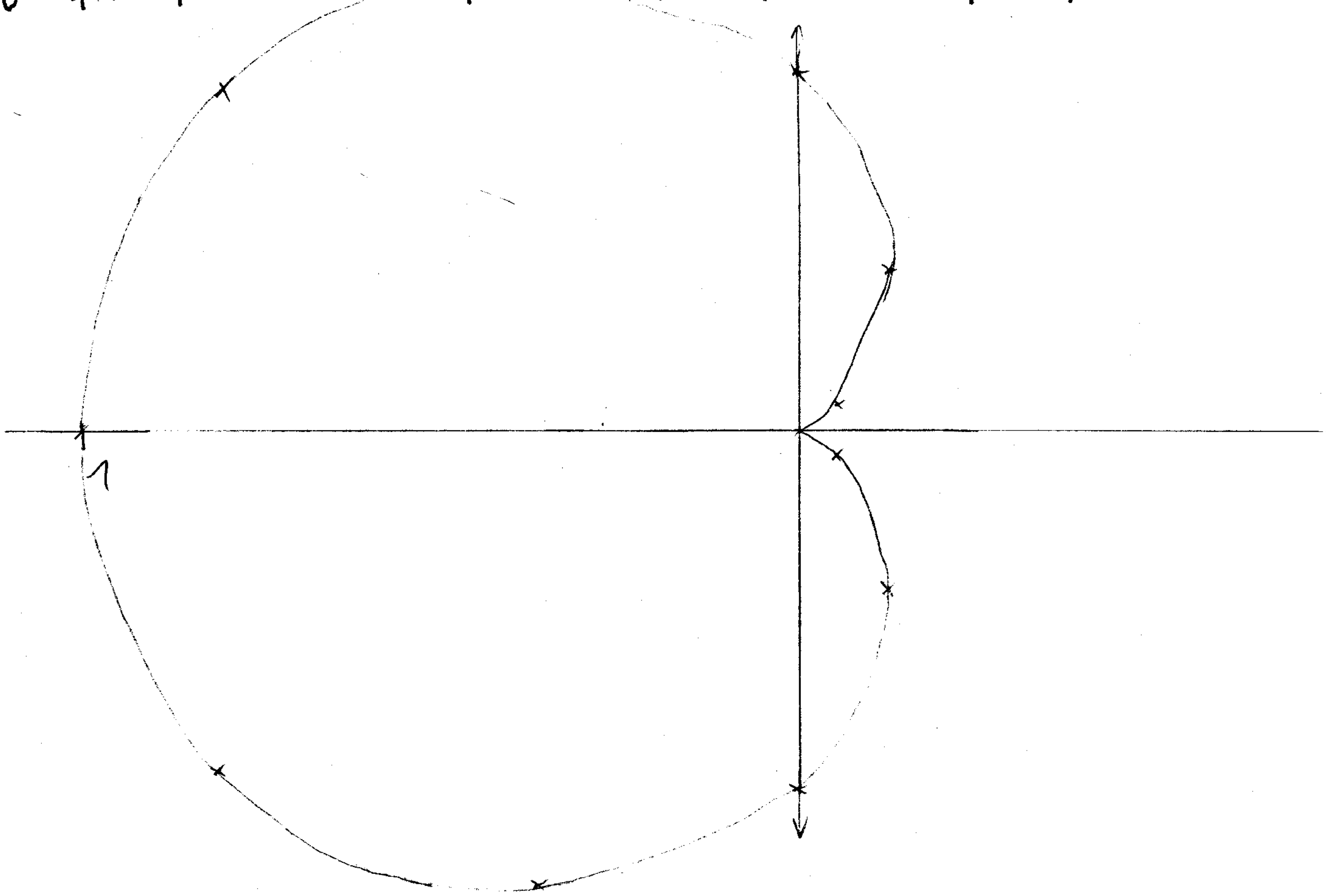
$\underline{\underline{A = 0{,}06794 \text{ FE}}}$

(43) $r(\varphi) = \sin^2\left(\frac{\varphi}{2}\right)$; $0 \leq \varphi \leq 2\pi$

Tipp: $\sin^4(\varphi) = \frac{1}{8} \cos(4\varphi) - 4\cos(2\varphi) + 3$
(vgl. $r(\varphi) = \cos^2\left(\frac{\varphi}{2}\right)$ auf nächste seite →)

Wertetabelle

0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	$\frac{6\pi}{6}$	$\frac{7\pi}{6}$	$\frac{8\pi}{6}$	$\frac{9\pi}{6}$	$\frac{10\pi}{6}$	$\frac{11\pi}{6}$	$\frac{12\pi}{6}$
0	0,07	0,25	0,5	0,75	0,93	1	0,93	0,75	0,5	0,25	0,07	0



$$A = \frac{1}{2}\int_0^{2\pi} r(\varphi)^2 \, d\varphi = \frac{1}{2}\int_0^{2\pi} \sin^4\left(\frac{\varphi}{2}\right) d\varphi = \frac{1}{2}\int_0^{2\pi} \frac{1}{8}\left(\cos\left(4\frac{\varphi}{2}\right) - 4\cos\left(2\frac{\varphi}{2}\right) + 3\right) d\varphi$$

$$A = \frac{1}{16}\int_0^{2\pi} (\cos(2\varphi) - 4\cos(\varphi) + 3) \, d\varphi = \frac{1}{16}\left[\int_0^{2\pi} \cos(2\varphi) \, d\varphi - 4\int_0^{2\pi} \cos(\varphi) \, d\varphi + 3\int_0^{2\pi} d\varphi\right]$$

$$A = \frac{1}{16}\left[\left(\frac{\sin(2\varphi)}{2} - 4\sin(\varphi) + 3\varphi\right)\Bigg|_0^{2\pi}\right] = \frac{1}{16}\left[\left(\frac{\sin(4\pi)}{2} - 4\sin(2\pi) + 6\pi\right) - \left(\frac{\sin(0)}{2} - 4\sin(0) + 0\right)\right]$$

$$A = \frac{1}{16} \cdot 6\pi = 1{,}178$$

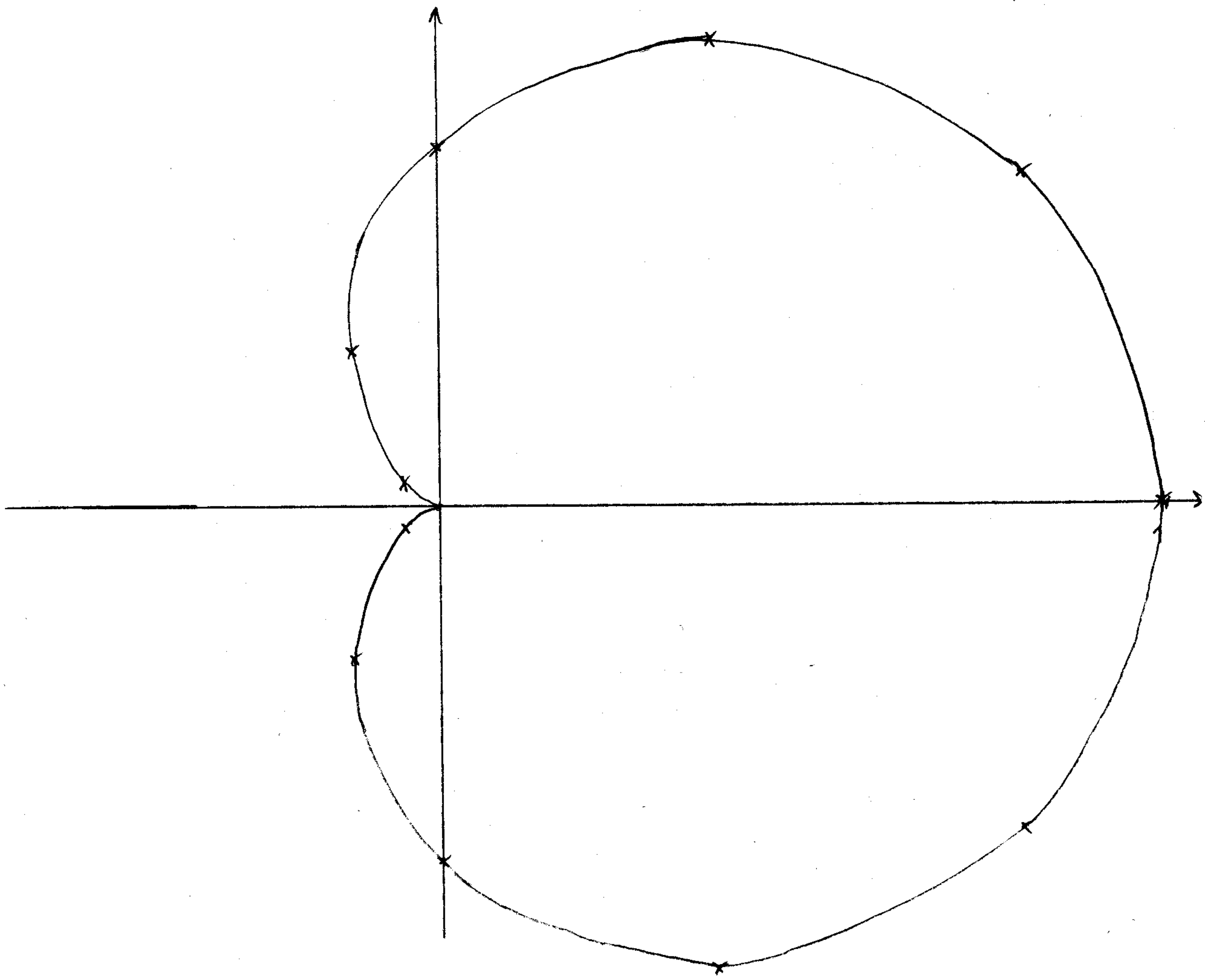
14/18.09.01

(13)

$r(\varphi) = \cos^2\left(\frac{\varphi}{2}\right)$ (vgl. $r(\varphi) = \sin^2\left(\frac{\varphi}{2}\right)$ auf vorheriger Seite)

$\alpha = \frac{\pi}{6}$

φ	0	α	2α	3α	4α	5α	6α	7α	8α	9α	10α	11α	12α
r	1	0,93	0,75	0,5	0,25	0,06	0	0,06	0,25	0,5	0,75	0,93	1



$$A = 2\frac{1}{2}\int_0^{\pi}\left[\cos^2\left(\frac{\varphi}{2}\right)\right]^2 d\varphi = \int_0^{\pi}\cos^4\left(\frac{\varphi}{2}\right)d\varphi = \frac{1}{8}\int_0^{\pi}\left[\cos(4\varphi) + 4\cos(2\varphi) + 3\right]d\varphi$$

$$A = \frac{1}{8}\left[\int_0^{\pi}\cos(4\varphi)\,d\varphi + \int_0^{\pi}4\cos(2\varphi)\,d\varphi + \int_0^{\pi}3\,d\varphi\right]$$

$4\varphi = t$

$\frac{dt}{d\varphi} = 4 \Rightarrow d\varphi = \frac{dt}{4}$

$$A = \frac{1}{8}\left[\frac{1}{4}\sin(t) + \frac{4}{2}\sin(s) + 3\varphi\right]_0^{\pi}$$

$2\varphi = s$

$\frac{ds}{d\varphi} = 2 \Rightarrow d\varphi = \frac{ds}{2}$

$$A = \frac{1}{8}\left[\underbrace{\frac{1}{4}\sin(4\cdot\pi)}_{0} + \underbrace{2\sin(2\pi)}_{0} + 3\pi - \underbrace{\frac{1}{4}\sin(0)}_{0} - \underbrace{2\sin(0)}_{0} - 0\right]$$

$$A = \frac{3\pi}{8}$$

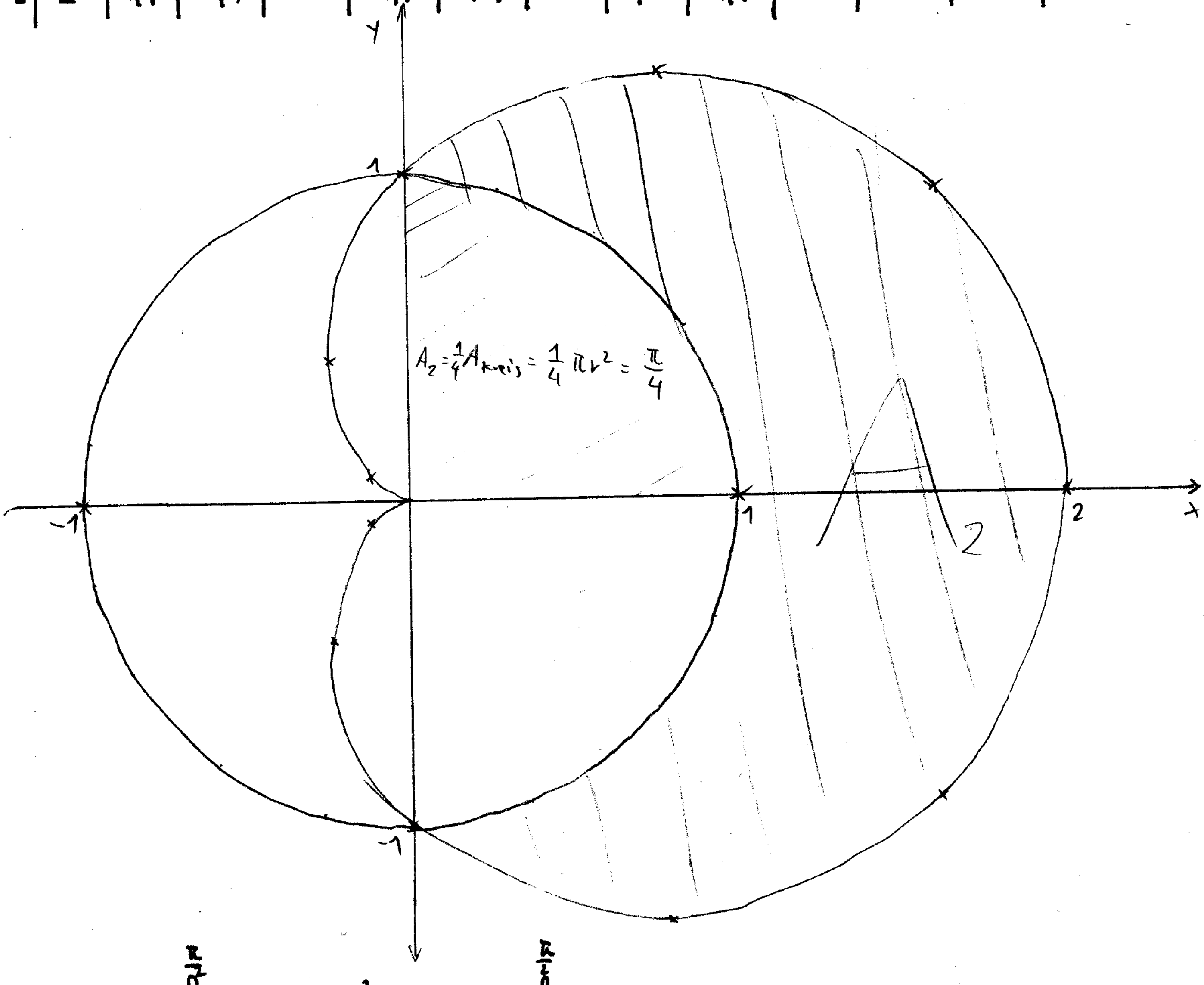
13.03.01 (13)

$\rho_1: \quad r(\varphi) = 1$

$\rho_2: \quad r(\varphi) = 1 + \cos(\varphi)$

$\alpha = \frac{\pi}{6}$

φ	0	α	2α	3α	4α	5α	6α	7α	8α	9α	10α	11α	12α
r_2	2	1,9	1,5	1	0,5	0,13	0	0,13	0,5	1	1,5	1,9	2



$$A_2 = A = 2\,\frac{1}{2}\int_0^{\frac{\pi}{2}} [1 + \cos(\varphi)]^2 \, d\varphi - \frac{\pi}{4} = \int_0^{\frac{\pi}{2}} 1 + 2\cos(\varphi) + \underbrace{\cos^2\varphi}_{\text{Typ 4}} \, d\varphi - \frac{\pi}{4}$$

$$A = \int_0^{\frac{\pi}{2}} d\varphi + 2\int_0^{\frac{\pi}{2}} \cos(\varphi)\, d\varphi + \int_0^{\frac{\pi}{2}} \cos^2(\varphi)\, d\varphi - \frac{\pi}{4}$$

$$A = \left[\varphi + 2\sin(\varphi) + \frac{1}{2}\varphi + \underbrace{\frac{1}{2}\sin\varphi \cdot \cos\varphi}_{\substack{0 \text{ für } \varphi = 0 \\ \text{und } \varphi = \frac{\pi}{2}}} \right]_0^{\frac{\pi}{2}} - \frac{\pi}{4} = \frac{\pi}{2} + 2 - \frac{\pi}{4} + \frac{1}{2}\frac{\pi}{2}$$

$$A = \frac{2}{4}\pi + 2 - \frac{\pi}{4} + \frac{1}{4}\pi = \frac{\pi}{2} + 2$$